Solutions to Review for the MA 153 Final

1. For positive or negative large values of $x$,
   
   
   $f(x) = 60 - 8x + 15x^2 + 25x^3 - 4x^4 + 40x^5 + x^6$
   
   looks like its leading term, the power function $y = x^6$.

   We can describe its long run behavior as follows:
   
   As $x \to -\infty$, then $y \to \infty$; as $x \to \infty$, then $y \to \infty$.

   Enlarge the viewing window to see that eventually the graph turns around.
   Choice B.

2. There are zeros at 0, 2, and 7. Therefore:
   
   $y = kt(t-2)(t-7)$

   $x = 1, \ y = -1 \Rightarrow -1 = k(1-2)(1-7)$

   $-1 = k(-6)$

   $k = \frac{1}{6}$

   The minimum value of $P(t)$ in the first ten seconds must be $P(10) = -40 \degree C$. This can be found using a graph or table or by evaluating $P(t) = -\frac{1}{6}t(t-2)(t-7)$ for $t = 10$.

   $P(10) = -\frac{1}{6}(10)(10-2)(10-7) = -\frac{1}{6}(10)(8)(3) = -40$.

   Choice D.

3. $Q(t) = -\frac{1}{6}t^3$ since remaining terms of lower degree
   
   Therefore, $P(t)$ and $Q(t)$ look very much alike for large values of $t$.
   (Note that the $-\frac{1}{6}$ is not optional.)

4. (a) Since all global behavior is shown, notice in the long run that graph $y_2$ is above graph $y_1$, which is above graph $y_3$.

   Exponential functions eventually outpace power functions, so the graph of $y = 4^t$ will be above the graphs of $y = 64x^2$ and $y = x^4$.

   Power functions with greater degree will outpace those of lower degree, so the graph of $y = x^4$ must eventually be above the graph of $y = 64x^2$.

   Extending the graphs, as shown to the right, may be helpful.

   Since $y_1 = 64x^2$, $y_2 = 4^x$, and $y_3 = x^4$ is not one of the choices, the correct answer to part (a) is E. None of these.

   (b) Enter all three equations in a grapher.

   To find the coordinates of point $P$, you need to find when the $y$-values of $Y2$ and $Y3$ are the same.

   (Don’t look at $Y1$ when trying to find the coordinates of point $P$.)

   If all three are highlighted in your $Y=$ menu they will all appear in your table.

   It might be easier to deselect $Y1$. Place your cursor on the equals sign for $Y1$ and press ENTER so $Y1$ is no longer highlighted.
Press 2nd WINDOW to make sure TblStart = 0 and ∆Tbl = 1.
Now press 2nd TABLE.
When \( x = 2 \) both \( y_2 = 4^x \) and \( y_3 = x^4 \) are 16.
When \( x = 4 \) both \( y_2 = 4^x \) and \( y_3 = x^4 \) are 256.

Which one of these is the point \( P \)?

Since we are given that the figure to the right shows all long run (global) behavior, the graph of \( y_2 = 4^x \) overtakes the graph of \( y_3 = x^4 \) at the point \( P \) and the graphs never intersect after that.

Thus the \( x \)-coordinate of \( P \) is the larger of 2 and 4, and \( P \) is the point \((4, 256)\).

Note: In addition to using a table, you can also solve the equation \( 4^x = x^4 \) using a graph with an Intersection feature on a graphing calculator. However, it can be clearer with a table to decide which solution is \( P \).
The equation \( 4^x = x^4 \) can not be solved using logarithms.

The point \( Q \) is where the graphs of \( y_1 = 64x^2 \) and \( y_3 = x^4 \) intersect as shown to the right.

In a similar way as before, we can use a table feature.
If you have all three formulas in your machine and scroll a table, ignore \( Y_2 \) when trying to find the coordinates of point \( Q \).

It can be easier to deselect \( Y_2 \) and examine only a table containing \( Y_1 \) and \( Y_3 \).

Now press 2nd TABLE.
When \( x = 0 \) both \( y_1 = 64x^2 \) and \( y_3 = x^4 \) are 0.
When \( x = 8 \) both \( y_1 = 64x^2 \) and \( y_3 = x^4 \) are 4096.
So \( Q \) has coordinates \((8, 4096)\).

You can also solve for this intersection point using the intersection feature with a graphing calculator, but what is even nicer is that you can solve the equation \( 64x^2 = x^4 \) algebraically by factoring. This will tell you all of the solutions that are possible.
(This can be helpful for part d later.) Set \( 64x^2 - x^4 = 0 \)
\[
x^2 (64 - x^2) = 0
\]
\[
x^2 = 0 \quad x^2 = 64
\]
\[
x = 0 \quad x = \pm 8
\]
Once you have the $x$-coordinate of $Q$, use a table or substitution to find the value of $y$ if $x = 8$. Thus $Q$ is the point $(8, 4096)$.

(c) We must find nonnegative values of $x$ which solve $y_1 \geq y_3$. Graphically, we find nonnegative values of $x$ for which the graph of $y_1 = 64x^2$ is above or intersects the graph of $y_3 = x^4$.

Because of part b, we know the graph of $y_1 = 64x^2$ and $y_3 = x^4$ intersect at $x = 0$ and $x = 8$ in the first quadrant.

Because of part a, we know the graph of $y_1 = 64x^2$ is above the graph of $y_3 = x^4$ from $x = 0$ and $x = 8$. Therefore for $x \geq 0$ the values of $x$ which solve $y_1 \geq y_3$ are those in the interval $[0, 8]$ or $0 \leq x \leq 8$.

(d) We must find all values of $x$ which solve $y_1 \geq y_3$. For any value of $x$, when is the graph of $y_1 = 64x^2$ above or intersects the graph of $y_3 = x^4$?

The objective here is to bring in what you know about power function shapes to solve a graphical inequality and be able to report it as an interval.

Both $y_1 = 64x^2$ and $y_3 = x^4$ are symmetric about the $y$-axis (even).

The solution is $[-8, 8]$ or $-8 \leq x \leq 8$.

You really don’t need a grapher to answer parts c and d if you know what the power function shapes are, use symmetry, and can solve the equation $64x^2 = x^4$ by getting $0$ on one side of the equation and factoring as shown in part b. Once you have those values of $x$ and use the concepts of the symmetry and know which power function overtakes the other, you can solve $y_1 \geq y_3$ graphically.

TIP: If you are trying to render these graphs in a calculator, first use the table to find a suitable window.

Recall the coordinates of the point $Q$ are $(8, 4096)$.

Set your Ymax to a value larger than 4096.

One possibility is $-10 \leq x \leq 10$ and $0 \leq y \leq 6000$. 

5. \( E(t) = 30t^{0.668} \). To find \( y = kt^p \), notice \( E(1) = 30 \) so if \( t = 1 \), then \( y = 30 \).

Therefore we have \( k = 30 \), since \( 30 = k(1)^p = k(1) = k \).

Now use another point to find \( p \) for \( y = 30t^p \). We used (2.02, 48).

\[
48 = 30(2.02)^p \\
\frac{48}{30} = (2.02)^p \\
1.6 = (2.02)^p \quad \text{So } p = \frac{\ln 1.6}{\ln 2.02} \approx 0.668.
\]

This means \( E(t) = 30x^{0.67} \) and \( E(7) = 30(7)^{0.67} \approx 110 \). Choice B.

6. \( S(t) = 5.61x^{1.37} \). To find \( y = kt^p \), use two points. We used (2.05, 15) and (2.98, 25).

\[
\frac{25}{15} = \frac{k(2.98)^p}{k(2.05)^p} \\
\frac{5}{3} = (\frac{2.98}{2.05})^p \\
p = \frac{\ln(5/3)}{\ln(2.98/2.05)} \approx 1.3655
\]

\( y = kt^{1.366} \) Now use any other point to find \( k \). We used (1.05, 6)

\[
k \approx 5.61 \\
S(t) = 5.61x^{1.37} \quad \text{Choice E.}
\]

7. \( E(t) = 30x^{0.67} \)

\[
S(t) = 5.61x^{1.37} \\
\frac{S}{E} = \frac{5.61x^{1.37}}{30x^{0.67}} = 0.187x^{0.7}
\]

Solve \( 0.187x^{0.7} > 0.75 \) by graphing \( y = 0.187x^{0.7} \) and the target line \( y = 0.75 \)

Perform an INTERSECTION routine or solve \( 0.187x^{0.7} = 0.75 \) to find the first time after which the ratio \( \frac{S}{E} \) is above 0.75. This is about 7.3 months. Choice D.

Window: \( 0 \leq x \leq 10 \), \( -0.25 \leq y \leq 1 \)
Note: You could also just enter

\[ Y_1:5.61x^{\frac{1.37}{30x^{.67}}} \]
\[ Y_2:0.75 \]

in a grapher, but the parentheses are crucial on a TI-83 or TI-83 Plus.

For example, you would NOT get the same function if you just typed

\[ Y_1:5.61x^{1.37}/30x^{.67} \]
\[ Y_2:0.75 \]

That would give you \( y = \frac{5.61x^{1.37}}{30} \cdot x^{0.67} \)

which is not what you want at all.

If you have a TI-84 or higher, use the fraction template \( \frac{n}{d} \) by pressing ALPHA Y=.

8. The town of Polynomia always exceeds 90 people.
   A population of 90 people = 0.9 hundred.
   Use a graphing calculator to sketch
   \( y = x^3 - 6x^2 + 8x + 4 \) and the line \( y = 0.9 \)
   in a viewing window such as
   \( 0 \leq x \leq 5 \) and \( 0 \leq y \leq 2 \)
   The polynomial never falls below the line.
   You could also use a calculator routine to determine
   the minimum value of the polynomial in this window,
   which is (3.1547, 0.920799).
   For \( t > 0 \) the lowest value of this town’s population is
   a mere 92 people!
   \( P(t) = t^3 - 6t^2 + 8t + 4 \) has the same long run behavior as \( y = x^3 \), so it will continually increase after \( t = 4 \) or
   after 1974. Since the town of Exponentia begins initially with 400 people and grows by 20\% each year, the
   formula for \( E(t) = 4(1.2)^t \). The graph of this function increases for all \( t \). Exponential functions will eventually
   outpace polynomial functions, so the graphs must cross more than three times. Using graphing technology,
   we can find that \( E(t) \) will intersect \( P(t) \) again about 58.88 years after 1970, or in the year 2028. To check this,
   sketch the difference function \( D(t) = E(t) - P(t) \) on a grapher and find when it is zero.
   The correct response is Choice E, all of the above are true.

9. We have \( C(t) = \frac{P(t)}{R(t)} = \frac{360 + 9t}{12,000 + 12t} \). Therefore \( C(0) = \frac{360 + 9(0)}{12,000 + 12(0)} = \frac{360}{12,000} = 0.03 \) or 3\%. Choice B.

10. As \( t \) gets larger and larger, the function \( C(t) = \frac{360 + 9t}{12,000 + 12t} \) approaches the ratio of the leading terms, namely
    \( \frac{9t}{12t} = 0.75 \). Eventually 75\% of the reservoir’s total volume would consist of pollutants. This can be confirmed
    with a graph of the function or a view of its table for large values of \( t \). Choice E.
11. a) The zeros of \( f(x) = 400x(6x^2 - 42) \) are 0, \( \sqrt{7} \), and \(-\sqrt{7}\).

Check graphically.
This third degree polynomial crosses the \( x \)-axis three times.
Find the zeros by solving \( f(x) = 0 \). Set each factor equal to 0 and solve.

\[
\begin{align*}
400x(6x^2 - 42) &= 0 \\
x &= 0 \\
6x^2 - 42 &= 0 \\
x^2 &= 7 \\
x &= \pm \sqrt{7}
\end{align*}
\]

b) The zeros of \( f(x) = -\frac{1}{100}(x - 2)(x^3 + 3x^2)(x + 1)^2 \) are 2, 0, \(-3\), \(-1\).

Set \( f(x) = 0 \) and factor completely:

\[
\begin{align*}
-\frac{1}{100}(x - 2)(x^3 + 3x^2)(x + 1)^2 &= 0 \\
-\frac{1}{100}(x - 2)x^2(x + 3)(x + 1)^2 &= 0 \\
-x^2(2x^2 - 5x + 2) &= 0 \\
2x^2(2x - 1)(x - 2) &= 0
\end{align*}
\]

Notice the graph crosses the \( x \)-axis four times, with double zeros at \(-1\) and 0 (where it bounces) and single zeros at \(-3\) and 2.

c) \( f(x) = 10x^3 - 4x^4 - 4x^2 \) has zeros at 0, \( \frac{1}{2} \), and 2.

\[
\begin{align*}
10x^3 - 4x^4 - 4x^2 &= 0 \\
-4x^4 + 10x^3 - 4x^2 &= 0 \\
-2x^2(2x^2 - 5x + 2) &= 0 \\
-2x^2(2x - 1)(x - 2) &= 0
\end{align*}
\]

Viewing a table or graph can help you factor.
12. Report the leading term, the leading coefficient, and the degree for each polynomial.

<table>
<thead>
<tr>
<th>Polynomial</th>
<th>leading term</th>
<th>leading coefficient</th>
<th>degree</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) $f(x) = 400x(6x^2 - 42)$</td>
<td>$2400x^3$</td>
<td>2400</td>
<td>2</td>
</tr>
<tr>
<td>= $2400x^3 - (a$ term of lower degree)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b) $f(x) = -\frac{1}{100}(x-2)(x^3 + 3x^2)(x+1)^2$</td>
<td>$-\frac{1}{100}x^6$</td>
<td>$-\frac{1}{100}$</td>
<td>6</td>
</tr>
<tr>
<td>= $-\frac{1}{100}x^6 + (terms$ of lower degree)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>c) $f(x) = 10x^3 - 4x^4 - 4x^2$</td>
<td>$-4x^4$</td>
<td>-4</td>
<td>4</td>
</tr>
</tbody>
</table>

The graphs of the leading terms share the same long run behavior as the graphs of $f(x)$ that are shown in the previous Question 11.

13. Report the formula of the power function $g(x)$ which has the same long run behavior as $f(x)$, graph $g(x)$, and, if it exists, report the equation of the horizontal asymptote of $f(x)$.

<table>
<thead>
<tr>
<th>Rational Function</th>
<th>$g(x)$</th>
<th>Graph of $g(x)$</th>
<th>Horizontal Asymptote</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) $f(x) = \frac{400x(6x^2 - 42)}{10x^3 - 4x^4 - 4x^2}$</td>
<td>$g(x) = \frac{-600}{x}$</td>
<td>$y = 0$</td>
<td></td>
</tr>
<tr>
<td>$\approx \frac{2400x^3}{-4x^4} = -\frac{600}{x}$ $\rightarrow 0 as x \rightarrow \pm \infty$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

If you enter $f(x)$ and $g(x)$ into a grapher, you could compare their tables and graphs for large values of $x$. Using the formula, however, can be much faster. Creating the table and the graph is not necessary to solve the problem.
Rational Function | \( g(x) \) | Graph of \( g(x) \) | Horizontal Asymptote
---|---|---|---
\( b) \ f(x) = \frac{400x(6x^2 - 42)}{10x - 4x^3 - 4x^7} \)
\approx \frac{2400x^3}{-4x^3} = -600 as \( x \to \pm \infty \) | \( g(x) = -600 \) | \( y = -600 \)

As before, creating the table and the graph is not necessary to solve the problem, but they are provided to show that the function \( f(x) \) looks like \( g(x) \) for large values of \( x \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( Y_1 )</th>
<th>( Y_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-25000</td>
<td>-600</td>
<td>-600</td>
</tr>
<tr>
<td>-20000</td>
<td>-600</td>
<td>-600</td>
</tr>
<tr>
<td>-15000</td>
<td>-600</td>
<td>-600</td>
</tr>
<tr>
<td>-10000</td>
<td>-600</td>
<td>-600</td>
</tr>
<tr>
<td>-5000</td>
<td>-600</td>
<td>-600</td>
</tr>
<tr>
<td>0</td>
<td>ERROR</td>
<td>600</td>
</tr>
<tr>
<td>5000</td>
<td>-599.9</td>
<td>-600</td>
</tr>
<tr>
<td>10000</td>
<td>-599.9</td>
<td>-600</td>
</tr>
<tr>
<td>15000</td>
<td>-600</td>
<td>-600</td>
</tr>
<tr>
<td>20000</td>
<td>-600</td>
<td>-600</td>
</tr>
<tr>
<td>25000</td>
<td>-600</td>
<td>-600</td>
</tr>
</tbody>
</table>

Rational Function | \( g(x) \) | Graph of \( g(x) \) | Horizontal Asymptote
---|---|---|---
\( c) \ f(x) = \frac{400x^2(6x^2 - 42)}{10x - 4x^3 - 4x^7} \)
\approx \frac{2400x^4}{-4x^3} = -600x as \( x \to \pm \infty \) | \( g(x) = -600x \) | None

The table and the graph are provided to show that the function \( f(x) \) looks like \( g(x) \) for large values of \( x \).
14. 30 lb of fertilizer produces a maximum yield of 400 pecks of peppers. Choice B.

15. Without applying any fertilizer at all, we see from the graph that the orchard will produce 175 pecks of peppers. Choice C.

16. The range is 0 \( \leq f(m) \leq 400 \). Note: You can also write [0, 400]. Choice E.

17. The function \( f(m) \) is increasing for \( 0 < m < 30 \). Choice C.
   Note: The function \( f(m) \) is decreasing for \( 30 < m < 70 \).

18. The function \( f(m) \) is never concave up and is concave down for \( 0 < m < 70 \). Choice E.

19. \( f(m) > 175 \) for \( 0 < m < 60 \).
   Determine where the graph of \( y = f(m) \) is above the line \( y = 175 \).
   The yield is more than 175 pecks of peppers when the amount of fertilizer applied is more than 0 lb and less than 60 lb. Choice D.

20. a) The equation of the axis of symmetry is the line \( x = 30 \). (Note: reporting just the number 30 is incorrect.) You could also report the equation of the line as \( m = 30 \).
   b) To find the vertex form, use a shift transformation of the graph of \( y = ax^2 \) (left 30 and up 400).
   We have \( y = a(x - 30)^2 + 400 \). Plug in the point (0, 175).
   
   \[
   y = a(x - 30)^2 + 400
   
   175 = a(0 - 30)^2 + 400 \\
   -225 = 900a \\
   a = -0.25
   \]

   In vertex form, \( f(x) = -0.25(x - 30)^2 + 400 \).

21. The function has a positive zero of 70, which is \( 70 - 30 = 40 \) units from the axis of symmetry.
   The other zero is also 40 units from the axis of symmetry or at \( 30 - 40 = -10 \).
   The factored form is \( y = a(x - 70)(x + 10) \), but from Question 20b, \( a = -0.25 \).
   In factored form, \( f(x) = -0.25(x - 70)(x + 10) \).

   Note: if we had not solved for \( a \) in Question 21, you can also find \( a \) by plugging in the point (0, 175).

   However, the leading coefficient \( a \) is the same for expanded form, vertex form, and factored form, so if you already have one of these formulas, you have \( a \).

22. \( P = 1160 + 10t \) and \( Q = 1000(1.0113)^t \)
   Set the equations equal to each other and solve using technology.
   They intersect at \( t = 39 \) years. Choice B.
23. \( Q = 20(0.4)^t = 20(1 - 0.6)^t \), so 60% of the drug is lost per hour. Choice \( \text{E} \).

24. The growth factor of \( y = ab^t \) is \( b \). Choice \( \text{A} \).

25. The equation is \( P = 9216(1.125)^t \). The initial amount when \( t = 0 \) is $9,216. Choice \( \text{C} \).

\[
\begin{align*}
\frac{a^{18}}{a^7} &= \frac{76787.03}{13122} \\
\frac{ab^{18}}{ab^7} &= \frac{76787.03}{13122} \\
\frac{\Delta b^{18}}{\Delta b^7} &= \frac{76787.03}{13122} \\
b^{18} &= \frac{76787.03}{13122} \\
\sqrt[18]{\frac{76787.03}{13122}} &= \left( \frac{76787.03}{13122} \right)^{1/18} = 1.125
\end{align*}
\]

\( b = \sqrt[18]{\frac{76787.03}{13122}} = \left( \frac{76787.03}{13122} \right)^{1/18} = 1.125 \)

\( P = 9216(1.125)^t \)

26. Since the equation is \( P = 9216(1.125)^t = 9216(1 + 0.125)^t \), the growth rate is 12.5%. Choice \( \text{C} \).

27. (a) We have been given that the equation is of the form \( y = ab^t + 60 \) and we must find \( a \) and \( b \).

When \( t = 0 \), \( y = 85 \) °F:

\[
85 = ab^0 + 60
\]

\[
85 = a + 60
\]

\[
a = 85 - 60 = 25.
\]

Note this is the initial temperature difference between the butler and the room temperature.

We have \( y = 25b^t + 60 \) and need \( b \).

When \( t = 2 \), \( y = 79.36 \) °F:

\[
79.36 = 25b^2 + 60
\]

\[
19.36 = 25b^2
\]

\[
b^2 = \frac{19.36}{25} = 0.7744
\]

\[
b = \sqrt{0.7744} = 0.88
\]

The model is \( y = 25(0.88)^t + 60 \). Check with a grapher or resubstitute the points. Choice \( \text{I} \).

(b) We must write \( y = 25(0.88)^t + 60 \) as \( y = He^{kt} + 60 \).

We can simplify this to writing \( 25(0.88)^t \) as \( He^{kt} \) for some constants \( H \) and \( k \).

The constant \( H \) is 25.

To find \( k \), set \( e^k = 0.88 \).

so \( k = \ln e^k = \ln(0.88) \)

To 3 decimal places, \( k = \ln (0.88) = -0.128 \) and we have \( y = 25e^{-0.128t} + 60 \).

Again we can check with a grapher or resubstitute the points. Choice \( \text{I} \).
(c) When his body temperature, \(y\), is 98.6ºF, we will assume the butler was alive. Set \(y = 25(0.88)^t + 60\) and \(y = 98.6\) equal to each other to find the time of death. (From the graph, we expect a negative number.)
Algebraic solution:
\[
25(0.88)^t + 60 = 98.6
\]
\[
25(0.88)^t = 38.6
\]
\[
(0.88)^t = \frac{38.6}{25}
\]
\[
\ln(0.88)^t = \ln\left(\frac{38.6}{25}\right)
\]
\[
t \ln(0.88) = \ln\left(\frac{38.6}{25}\right)
\]
\[
t = \frac{\ln\left(\frac{38.6}{25}\right)}{\ln(0.88)} \approx -3.4
\]

Notice the timeline on the graph:
He died 3.4 hours before 6:00 pm.
This is 3 hours and 0.4 \(\times\) 60 = 24 minutes before 6:00 pm
(or 24 minutes prior to 3:00 pm) which is 2:36 pm. Since the house records indicate that the niece arrived at 2:45 pm., the butler was already dead when she arrived.

Note: You could have also have used the equation involving \(e\) as shown below.
Since \(\ln(0.88) = -0.128\), you reach the same answer:
\[
25e^{-0.128t} + 60 = 98.6
\]
\[
25e^{-0.128t} = 38.6
\]
\[
e^{-0.128t} = \frac{38.6}{25}
\]
\[
\ln e^{-0.128t} = \ln\left(\frac{38.6}{25}\right)
\]
\[
-0.128t = \ln\left(\frac{38.6}{25}\right)
\]
\[
t = \frac{\ln\left(\frac{38.6}{25}\right)}{-0.128} \approx -3.4
\]

28. (a) It might be helpful to plot the points and organize the information in a table.

<table>
<thead>
<tr>
<th>(w)</th>
<th>(C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>$32</td>
</tr>
<tr>
<td>180</td>
<td>$48</td>
</tr>
</tbody>
</table>

The slope is positive:
\[
m = \frac{\Delta C}{\Delta w} = \frac{48 - 32}{180 - 100} = \frac{16}{80} = 0.2
\]
We have \(C = b + 0.2w\).
Substitute \(w = 100\), \(C = $32\):
\[
32 = b + 0.2(100)
\]
\[
32 = b + 20
\]
\[
b = 12
\]

Therefore \(C = 12 + 0.2w\).

(b) The slope is $0.20 per kg, which is the monthly rate that the service charges for waste collection.

(c) The vertical intercept is (0, $12). When no waste is collected, the service charges a fixed charge of $12.

29. (a) Since we start with 900 gallons of fresh water, the vertical intercept is (0, 900).
Each day we lose 12 gallons of water so the equation is \(f(t) = 900 - 12t\).

(b) (i) \(f(0) = 900\).
Initially we have 900 gallons of water.
(ii) To find \( f^{-1}(0) = t \), we must find the time \( t \) when the team has 0 gallons of fresh water.

\[
\begin{align*}
0 &= 900 - 12t \\
12t &= 900 \\
t &= \frac{900}{12} = 75
\end{align*}
\]

Thus \( f^{-1}(0) = 75 \) days.

It will take 75 days before the team has 0 gallons of water remaining.

30. (a) When we have zero U.S. dollars, we have zero shillings: the \( y \)-intercept is \((0, 0)\).

(b) We need an equation for \( y = f(x) \).

We first find the slope of the function.

The function is increasing so we expect a positive slope.

One way is to find the slope is to compute \( \Delta y \) and \( \Delta x \).

We want \( \frac{\Delta y}{\Delta x} = \frac{3300 \text{ shillings}}{\$1.50} = 2200 \)

Check that this is also the same as \( \frac{1100 \text{ shillings}}{\$0.50} = 2200 \text{ shillings per U.S. dollar} \).

Since the \( y \)-intercept is \((0, 0)\), the equation is \( y = 2200x \).

Check: The equation passes through the point \((1, 2200)\), as well as the other points in the table.

If \( y = 4000 \text{ shillings} \), then \( 4000 = 2200x \) so \( x = \frac{4000}{2200} \approx \$1.82 \). The trousers cost \$1.82.

(Recall these are second-hand items in the Kampala market.)

(c) If we have \( x = \$4.00 \), then we can exchange it for \( y = 2200x = 2200 \cdot 4 = 8800 \text{ shillings} \), so we can afford the \$8500 \text{ shilling coat}.

31. (a) We know that when the price \( p = \$10 \), the number of customers \( N \) who will come to the park is 10,000. For each \$1.00 increase in the entrance price \( p \), the park would lose an average of 500 daily customers:

<table>
<thead>
<tr>
<th>( p )</th>
<th>( N )</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10.00</td>
<td>10,000</td>
</tr>
<tr>
<td>$11.00</td>
<td>9,500</td>
</tr>
<tr>
<td>$12.00</td>
<td>9,000</td>
</tr>
</tbody>
</table>

(b) \( N = f(p) \) is linear. When \( \Delta p = 1 \), then \( \Delta N = -500 \).

The slope is \( \frac{\Delta N}{\Delta p} = \frac{500}{\$1} = -500 \) and it passes through \((10, 10,000)\).

We have \( N = b - 500p \). Substitute \( p = 10 \), \( N = 10,000 \):

\[
egin{align*}
10,000 &= b - 500(10) \\
10,000 &= b - 500 \\
b &= 15,000
\end{align*}
\]

Therefore \( N = f(p) = 15,000 - 500p \). \( TIP \): Check the formula using the table feature of a grapher.

(c) If 10,000 customers pay \$10 each, the revenue is 10,000 \cdot \$10 = \$100,000. Do this for each row to complete the table. Notice revenue increases due to the ticket price increase, although \( N \) decreases.

<table>
<thead>
<tr>
<th>( p )</th>
<th>( N )</th>
<th>( R = p \cdot N )</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10.00</td>
<td>10,000</td>
<td>$100,000</td>
</tr>
<tr>
<td>$11.00</td>
<td>9,500</td>
<td>$104,500</td>
</tr>
<tr>
<td>$12.00</td>
<td>9,000</td>
<td>$108,000</td>
</tr>
<tr>
<td>$13.00</td>
<td>8,500</td>
<td>$110,500</td>
</tr>
<tr>
<td>$14.00</td>
<td>8,000</td>
<td>$112,000</td>
</tr>
</tbody>
</table>

(d) In general, \( R \) is the product of the first two columns, so \( R = p \cdot N \). Since \( N = 15,000 - 500p \), we have \( R = p \cdot N = p \cdot (15,000 - 500p) \).

Check the formula using the table feature of a grapher.

(e) To find the \( N \)-intercept of \( N = 15,000 - 500p \), set \( p = 0 \). By inspection it is \((0, 15,000)\).

Interpretation: If the tickets were free, the amusement park would have 15,000 customers.

To find the \( p \)-intercept of \( N = f(p) \), set \( N = 0 \) and solve for \( p \):

\[
0 = 15000 - 500p \Rightarrow 500p = 15000 \Rightarrow p = 30
\]

The \( p \)-intercept is \((30, 0)\).

Interpretation: If the ticket price was \$30, no customer would purchase one.
(f) Find any p-intercepts by solving \( R = p \cdot (15,000 - 500p) = 0 \)
Set each factor equal to 0: we know \( R = 0 \) when \( p = 0 \) and \( 15,000 - 500p = 0 \).
From part (e), \( 15,000 - 500p = 0 \) when \( p = 30 \) so the p-intercepts are (0, 0) and (30, 0).
Interpretation:
(0, 0): If the tickets were free, there would be no revenue (even though 15000 customers would come).
(30, 0): If the tickets were $30, there would be no revenue (since no customers would buy them.)

To find all the R-intercepts, set \( p = 0 \) in the equation \( R = g(p) = p \cdot (15,000 - 500p) \).
\( R = g(0) = 0 \cdot 15,000 = 0 \).
The only \( R \)-intercept is the point (0, 0), interpreted previously.

(g) $15 since the maximum is at \((15, 112,500)\).
There are several strategies to get the maximum of \( R(x) = x(15,000 - 500x) = -500x^2 + 15000x \)
1. The coefficient of the \( x^2 \) term is negative, so the parabola is concave down. Since a parabola is symmetric about its maximum, and its zeros are at \( x = 0 \) and 30, the maximum is midway between at \( x = 15 \). The \( y= \) coordinate of the maximum is 
   \( R = g(15) = 15 \cdot (15,000 - 500 \cdot 15) = 112,500 \).
2. Use the maximum feature.
3. Use the table feature.
(h) See the graph at the right.

32. \( e^{\ln a} = e^{\ln a} = a^x \). Choice E.

33. (a) Choice III  (b) Choice VI  (c) Choice IV  (d) Choice IV  (e) Choice I

(f) Choice III  (g) Choice II  (h) Choices I and III

(i) Choices I, II, and VI  (j) Choice VII  Choices V, VI, and VII
34. a. Choice II. The train’s speed slows to a stop (speed is 0).

b. Choice I. My rate is constant at first, so the graph appears linear. Once the chimes ring, my rate increases so the graph is concave up.

c. Choice III. First my speed is constant, or flat. The graph appears horizontal. When I run, my speed increases.

d. Choice II. The ferris wheel car climbs to its highest point, then descends, then climbs again.

e. Choice III. As the child climbs up the slide her speed is steady and constant. When she stops at the top of the slide, her speed is 0. Once she slides down her speed increases, exceeding the speed she had when she was climbing the slide. At the bottom of the slide, her speed is 0 when she stops.

35. (i) \( P(t) = 300 - 2t \) is Choice F since \( 300 - 2t = 250 \) when \( t = 25 \). The population starts at 300 and has dropped to 250 after 25 years. It is not Choice A since, even though \( P(t) = 300 - 2t \) declines at a constant rate, \( P(t) \) becomes 0 in 150 years, not 15.

(ii) \( Q(t) = 300e^{-0.02t} \) is Choice C. The population, which began at 300, is growing at the continuous rate of 2 percent each year.

(iii) \( R(t) = 300(0.98)^t \) is Choice H. The population, originally at 300, has been decreasing at the annual rate of 2 percent.

(iv) \( S(t) = -\frac{4}{9}t^2 + 300 \) is Choice G. The population, which began at 300, decreases faster and faster.

36. a) Factor \( h(x) \) to find the zeros. \( 0.96x - 0.004x^2 = 0 \)

\[
\begin{align*}
  x(0.96 - 0.004x) &= 0 \\
  x &= 0 \\
  0.96 - 0.004x &= 0 \\
  0.004x &= 0.96 \\
  x &= \frac{0.96}{0.004} = 240
\end{align*}
\]

\( h(x) \) has zeros at 0 and 240 and is concave down since \( a = -0.004 \). Use a table to find the vertex, which is halfway between the zeros at the point (120, 57.6) so the exact maximum height is 57.6 ft.

b) The shell hits the ground 240 feet from the base.

c) The vertex is (120, 57.6).

d) The equation of the axis of symmetry is \( x = 120 \). Note: reporting just 120 is not correct.

e) In vertex form \( h(x) = a(x - 120)^2 + 57.6 \), but since \( h(x) = -0.004x^2 + 0.96x \), the value of \( a = -0.004 \). So \( h(x) = -0.004(x - 120)^2 + 57.6 \)

f) In factored form \( h(x) = -0.004(x - 240) \). Note: Although \( h(x) = x(0.96 - 0.004x) \) is also in factored form, the equation \( h(x) = -0.004(x - 240) \) has the advantage of showing the zeros.
37. (a) The horizontal asymptote is \( y = 4 \). The vertical asymptotes are \( x = 0 \) and \( x = 2 \).

To find if there is a horizontal asymptote, examine the long run behavior:

\[
y = \frac{8x^2 - 8}{2x^2 - 4x} \rightarrow \frac{8x^2}{2x^2} = 4 \quad \text{as} \quad x \rightarrow \pm \infty
\]

Since the function looks like the line \( y = 4 \) for very large values of \( x \), the line \( y = 4 \) is the horizontal asymptote.

To find the vertical asymptotes, factor the denominator:

\[
y = \frac{8x^2 - 8}{2x^2 - 4x} = \frac{8(x^2 - 1)}{2x(x - 2)} = \frac{4(x - 1)(x + 1)}{x(x - 2)}
\]

The function has vertical asymptotes when the denominator is zero (and the numerator is not).

The denominator \( x(x - 2) = 0 \) when \( x = 0 \) and \( x = 2 \).

(b) We can find the domain of \( f(x) = \sqrt{x - 100} \) using the graph, the table or reason from the formula.

The graph of \( f(x) = \sqrt{x - 100} \) is a horizontal shift of the graph of the power function \( y = \sqrt{x} \) right 100 units.

The domain is \( x \geq 100 \).

You can also write the domain \( [100, \infty) \).

Use a table to confirm that 100 is included in the domain, as well as all reals larger than 100.

Values less than 100 cause the calculator to bail.

The formula \( f(x) = \sqrt{x - 100} \) tells you that \( f(x) \) is defined if the radicand \( x - 100 \geq 0 \).

When you solve this inequality, you have \( x \geq 100 \).
(c) If a substance decays according to the formula \( P(t) = 200(0.5)^{t/17} \), where \( t \) is in minutes, its half-life is 17 minutes. Check by substitution. To find the percent of the substance which decays each minute, first find the growth factor.

Thus \( P(t) = 200(0.5)^{t} = 200(0.5^{17})^t = 200(0.96)^t \).
Since 96% of the substance is retained each minute, we have that 4% decays each minute.

(d) If a population with initial amount \( P_0 \) doubles every 12 years, it is modeled by \( P(t) = P_0(2)^{t/12} \).

To find the tripling time, solve \( P(t) = P_0(2)^{t/12} = 3P_0 \).

Divide both sides by \( P_0 \) and take logarithms:
\[
(2)^{t/12} = 3
\]
\[
\log(2)^{t/12} = \log 3
\]
\[
\frac{t}{12} \log(2) = \log 3
\]
\[
t = \frac{12 \log 3}{\log 2} \approx 19 \text{ years}.
\]

We can check by substituting back into the original equation. \( P(19) = P_0(2)^{19/12} \approx 3P_0 \).

(e) \( x = 2 \) is a solution to the equation \( 4x + 8 = 4^x \) since

\[
4x + 8 = 4^x
4(2) + 8 = 4^2
8 + 8 = 16
\]

To solve \( 4x + 8 > 4^x \), we must find all solutions to \( 4x + 8 = 4^x \), which can only be solved graphically or numerically.
The solutions are \( x = -1.98403 \) and \( x = 2 \).

From the graph, the solution to \( 4x + 8 > 4^x \) are the values of \( x \) when the graph of \( y = 4x + 8 \) is above the graph of \( y = 4^x \), which is \(-1.98403 < x < 2 \)

(f) False. For example: Since \( \log 10 = 1 \), we have \( \log 10 + \log 10 = 1 + 1 = 2 \).
But \( 2 = \log 10^2 = \log 100 \), not \( \log 20 \).
Thus \( \log (10 + 10) = \log 20 \) is not equal to \( \log 10 + \log 10 = 2 = \log 100 \).

Note: \( \log A + \log B = \log(AB) \)
\( \log A + \log B \neq \log(A + B) \)

38. Choice C. \( h(x) = x^3 \) The domain and range of \( h(x) = x^3 \) are all real numbers.
39. (a) The polynomial has formula \( y = \frac{1}{4}(x-2)(x-1)(x+3)(x+2)^2 \)
Because the function has single zeros at \(-3\), \(1\), and \(2\) and a double zero at \(-2\) we can write \( y = a(x-2)(x-1)(x+3)(x+2)^2 \) Now substitute the point \((0,6)\):

\[
x = 0 \quad \Rightarrow \quad y = a(x-2)(x-1)(x+3)(x+2)^2
\]

\[
y = 6 \quad \Rightarrow \quad 6 = a(-2)(-1)(3)(2)^2
\]

\[
a = \frac{6}{24} = \frac{1}{4}
\]

Therefore, the polynomial is \( f(x) = \frac{1}{4}(x-2)(x-1)(x+3)(x+2)^2 \)
To find \( f(3) \), let \( x = 3 \):

\[
f(3) = \frac{1}{4}(3-2)(3-1)(3+3)(3+2)^2 = \frac{1}{4}(1)(2)(6)(5)^2 = 75
\]

You could also use the table feature of a graphing calculator. Choice B.

**Important:**
You should check with a graphing calculator to be sure that the function is correct.

(b) The rational function has the formula \( y = \frac{4(x-2)}{(x-3)} \)
Because the zeros of the function is 2, we have \((x-2)\) as a factor of the numerator since the function is 0 when the numerator is 0.

Since the vertical asymptote is \(x = 3\), we have \((x-3)\) as a factor of the denominator.
(The vertical asymptotes are found where the denominator is 0 and the numerator is not).

So we can write \( y = \frac{a(x-2)}{(x-3)} \).

Since the horizontal asymptote is \(y = 4\) and it is found by the ratio of the leading terms, we must have \(a = 4\).

Therefore the function must be \( f(x) = \frac{4(x-2)}{(x-3)} \). Use a table feature to find Choice C is correct.

Alternatively, use the formula: \( f(403) = \frac{4(403-2)}{(403-3)} = \frac{4 \cdot 401}{400} = \frac{401}{100} = 4.01 \)

\( f(403) = 4.01 \)
(c) The rational function has the formula $y = \frac{2x(x+3)}{(x+2)^2}$.

Because the zeros of the function are 0 and −3, the factors of the numerator are $x(x+3)$, since the function is 0 when the numerator is 0.

There is one vertical asymptote at $x = -2$, so $(x + 2)$ is a factor of the denominator. However, the short run behavior near this asymptote looks like $y = k/x^2$ ( ) so the factor must have a power of 2.

We can write $y = \frac{ax(x+3)}{(x+2)^2}$. Since the horizontal asymptote is $y = 2$, we must have $a = 2$.

Note: $y = \frac{ax(x+3)}{(x+2)^2} \approx \frac{ax^2}{x^2} = a$ as $x \to \pm\infty$ so $a = 2$.

Therefore, the rational function has the formula $y = \frac{2x(x+3)}{(x+2)^2}$.

Use a table to confirm:

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-6</td>
<td>2.25</td>
</tr>
<tr>
<td>-5</td>
<td>2.2222</td>
</tr>
<tr>
<td>-4</td>
<td>2</td>
</tr>
<tr>
<td>-3</td>
<td>0</td>
</tr>
<tr>
<td>-2</td>
<td>ERROR</td>
</tr>
<tr>
<td>-1</td>
<td>-4</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0.8889</td>
</tr>
<tr>
<td>2</td>
<td>1.25</td>
</tr>
<tr>
<td>3</td>
<td>1.44</td>
</tr>
<tr>
<td>4</td>
<td>1.5556</td>
</tr>
</tbody>
</table>

This should match the given information provided.

Use a table to find determine if $f(-1) = -4$, $f(1) = 1$, and $f(-6) = 2.25$:

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-6</td>
<td>2.25</td>
</tr>
<tr>
<td>-5</td>
<td>2.2222</td>
</tr>
<tr>
<td>-4</td>
<td>2</td>
</tr>
<tr>
<td>-3</td>
<td>0</td>
</tr>
<tr>
<td>-2</td>
<td>ERROR</td>
</tr>
<tr>
<td>-1</td>
<td>-4</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0.8889</td>
</tr>
<tr>
<td>2</td>
<td>1.25</td>
</tr>
<tr>
<td>3</td>
<td>1.44</td>
</tr>
<tr>
<td>4</td>
<td>1.5556</td>
</tr>
</tbody>
</table>

Since only Choices A and C are true, Choice D is correct.

40. The equation is $y = \frac{8(x-4)}{(x-2)^2}$.

Because there is a horizontal asymptote of $y = 0$, the degree of the numerator is less than the degree of the denominator. The numerator has a factor of $(x-4)$ since it has a single zero. Because the short run behavior near the vertical asymptote looks like $y = k/x^2$ or $y = k/x^3$, the lowest degree possible for the denominator must be 2.
So it has a factor of \((x - 2)^2\). It has the form \(y = \frac{a(x - 4)}{(x - 2)^2}\), and we can find \(a\) if we use the fact that when \(x = 0, y = -8\):

\[
-8 = \frac{a(0 - 4)}{(0 - 2)^2} \\
-8 = \frac{4}{a} \\
a = 8
\]

So \(f(x) = \frac{8(x - 4)}{(x - 2)^2}\). To find \(f(3)\), we let \(x = 3\) and find \(y\).

\[
f(3) = \frac{8(3 - 4)}{(3 - 2)^2} = \frac{8(-1)}{1} = -8
\]

Alternatively, you can enter the formula in a grapher and use a table. Choice B.

41. The degree of the factor \((x - a)\) must be even since there is a bounce at the zero.

The degree of the factor \((x - b)\) must be even since the vertical asymptote appears as near \(b\).

The degree of the factor \((x - c)\) must be 3, 5, … since there is a chair at the zero.

The degree of the factor \((x - d)\) must be even since the vertical asymptote appears as near \(d\).

The long run behavior is the same as the power function \(y = kx\), so the degree of the numerator must be one more than the degree of the denominator. Therefore, it must be Choice B.

42. I. Choice C. \(y = B - Ax\) since it has a positive \(y\)-intercept \((B)\) and slope is negative \((-A)\).

II. Choice C \(y = \log(x + A)\) since it is a shift of \(y = \log x\) to the left \(A\) units. (Its vertical asymptote is at \(x = -A\).)

III. Choice A \(y = |x - A|\) since it is a shift of \(y = |x|\) to the right \(A\) units. (Its minimum is when \(x = A\).)

IV. Choice C \(y = A(x + B)^2 - C\) since the \(x\)-and \(y\)-coordinate coordinates of the vertex are negative and the parabola is concave up.

V. Choice C \(y = -A(x + B)^5 + C\) since it is a vertical reflection of \(y = x^5\) combined with a horizontal shift to the left and a vertical shift up.

VI. Choice D \(y = (1 / A)^x\) since it is exponential decay.
VII. Choice C \( y = \frac{A(x + B)}{x - C} \) since its vertical asymptote is \( x = C \) with \( C \) positive, it has a horizontal asymptote \( y = A \) with \( A \) positive, and a negative zero (at \(-B\)).

VIII. Choice A \( y = \frac{A}{(x - B)^2} - C \) since it is a shift of \( y = \frac{A}{x^2} \) to the right \( B \) units and down \( C \) units.

43. The average rate of change is \( \frac{\Delta V}{\Delta t} \).

<table>
<thead>
<tr>
<th>Time, ( t ) (min)</th>
<th>Volume, ( V ) (gal)</th>
<th>( \Delta V )</th>
</tr>
</thead>
<tbody>
<tr>
<td>30 min</td>
<td>30</td>
<td>1075</td>
</tr>
<tr>
<td>30 min</td>
<td>60</td>
<td>1150</td>
</tr>
<tr>
<td>30 min</td>
<td>90</td>
<td>1225</td>
</tr>
<tr>
<td>30 min</td>
<td>120</td>
<td>1300</td>
</tr>
</tbody>
</table>

Find the change in time, \( \Delta t \), and the change in volume, \( \Delta V \), over the intervals.

Then create ratios. The average rate of change will be the rate at which the water fills the pool.

\[ \frac{\Delta V}{\Delta t} = \frac{75 \text{ gal}}{30 \text{ min}} = 2.5 \text{ gallons per minute.} \]

The answer is Choice E.

44. The range is the set of all possible values of \( y \).

The range of the function \( y = 5x^2 \) is all real numbers greater than equal to 0.

The function shown is a translation of \( y = 5x^2 \) up 1, so the range is \([1, \infty)\).

Notice on the graph to the right, values of \( y \) begin at \( y = 1 \) and increase forever.

Choice B.

45. \[ \log_b \left( \frac{x^3 y^2}{\sqrt{w}} \right) = \log_b x^3 + \log_b y^2 - \log_b \sqrt{w} \]

\[ = \log_b x^3 + 2 \log_b y - \frac{1}{2} \log_b w \]

The correct answer is Choice C.

46. \( 25^x = 3^{600} \)

\[ \ln 25^x = \ln 3^{600} \]

\[ x \ln 25 = 600 \ln 3 \]

\[ x = \frac{600 \ln 3}{\ln 25} \approx 204.78 \]

The correct answer is Choice C.

47. \((-1, 96)\) You can find the vertex by completing the square or use technology. Choice D.

**Note:** If you use technology to find the minimum of the graph, you would not report points such as \((-1.000001, 96)\) or \((-0.9999983, 96)\).

The table feature, however, would show \((-1, 96)\).
48. Solve \(4,000e^{0.073t} = 12,000\).

Divide both sides by 4000 to get \(e^{0.073t}\) all by itself.

Take natural logarithms of both sides.

Use the inverse property: \(\ln e^{0.073t} = 0.073t\).

Divide both sides by 0.073 to solve for \(t\).

\[ t = \frac{\ln 3}{0.073} \approx 15.05 \]

Choice D. TIP: Check by resubstituting:

\[ 4000e^{0.073 \times 15.05} \approx 12000 \]

49. Sketch a graph of the polynomial \(f(x) = 9x^2(x + 6)(x - 6)^2\) by hand (or use a grapher, but it’s difficult to find a window). Determine the values of \(x\) for which \(f\) is above or on the \(x\)-axis, which is \(x \geq -6\).

The correct answer is Choice C.

50. Solve \(4,000e^{0.073t} = 12,000\).

\[ 4000e^{0.073t} = 12,000 \quad \text{Divide both sides by 4000 to get} \quad e^{0.073t} \quad \text{all by itself.} \]

\[ e^{0.073t} = 3 \quad \text{Take natural logarithms of both sides.} \]

\[ \ln e^{0.073t} = \ln 3 \quad \text{Use the inverse property:} \quad \ln e^{0.073t} = 0.073t. \]

\[ 0.073t = \ln 3 \quad \text{Divide both sides by 0.073 to solve for} \quad t. \]

\[ t = \frac{\ln 3}{0.073} \approx 15.05 \]

Choice D. TIP: Check by resubstituting:

\[ 4000e^{0.073 \times 15.05} \approx 12000 \]

51. \(\ln \left( \frac{1}{\sqrt{e^x}} \right) = \ln \left( \frac{1}{e^{x/2}} \right) = \ln (e^{-x/2}) = -\frac{x}{2}\) Choice C.

52. In general, the graph of \(y = f(x) = ab^x\) increases for \(b > 1\) and decreases for \(0 < b < 1\) and has \(y\)-intercept \((0, a)\).

The function \(y = b^x\) is a special case, with \(a = 1\). Therefore, Items I and III are correct. Choice D.
53. The graph of \( y = 2 + \log(x - 1) \) is a horizontal shift 1 unit to the right and a vertical shift 2 units up of the graph of \( y = \log(x) \).

- Since the graph of \( y = \log(x) \) has a vertical asymptote of \( x = 0 \), the graph of \( y = 2 + \log(x - 1) \) has a vertical asymptote of \( x = 1 \).
- Since the domain of \( y = \log(x) \) is the set of all real numbers \( x > 0 \), the domain of \( y = 2 + \log(x - 1) \) is the set of all real numbers \( x > 1 \). Therefore it does not cross the \( x \)-axis at 1 and it never touches the \( y \)-axis.
- The graph of \( y = 2 + \log(x - 1) \) passes through the point (2, 2): check: \( x = 2, \ y = 2 \Rightarrow y = 2 + \log(2 - 1) \)
  \[ 2 = 2 + \log(2 - 1) \]
  \[ 2 = 2 + \log(1) \]
  \[ 2 = 2 + 0 \]  YES

- The range of the function \( y = 2 + \log(x - 1) \) is all real numbers.

It is difficult for most technology to produce an accurate graph of a logarithm function. Don’t be deceived by a misleading graph.

Therefore Items I, III, and IV are correct. Choice E.

54. Since the vertical asymptote is \( x = a \), the denominator must have \( (x - a) \) as a factor.

Since the function has a single zero through the origin \( (0, 0) \), the numerator must be 0 when \( x = 0 \).

The short run behavior of the function near its vertical asymptote looks like requiring the factor in the denominator to be raised to an odd power.

The equation \( y = \frac{x}{x-a} \) is the only choice which meets these three criteria. Choice C.

55. As \( x \to \infty \) or \( x \to -\infty \), \( f(x) = \frac{2ax}{(x-a)^2} \approx \frac{2ax}{x^2} = \frac{2a}{x} \).

In other words, the graph of \( y = \frac{2ax}{(x-a)^2} \) and the graph of \( y = \frac{2a}{x} \) have the same long run behavior. The graph of \( y = \frac{2a}{x} \) has end behavior which looks like \( \frac{1}{x} \) or \( \frac{1}{x} \) (depending on whether \( a \) is positive or negative).

In either case, as \( x \to -\infty \) or as \( x \to \infty \), the function approaches 0. The horizontal asymptote is \( y = 0 \). Choice D.

56. Since \( \text{pH} = -\log C \) and \( \text{pH} = 2.1 \), we must solve the logarithmic equation

\[
2.1 = -\log C
\]

\[
-\log C = 2.1
\]

\[
\log C = -2.1
\]

\[
10^{-2.1} = 10^{0.1} \approx 0.0008
\]

Multiply both sides by \(-1\)

Make both sides a power of 10

Use an inverse property

\[ \text{Normal Float Auto Real Degree HP} \]

\[ 10^{-2.1} \approx 0.0079432823 \]

\[ -\log(\text{Ans}) \approx 2.1 \]

Choice B.
57. To solve $\ln 2x^3 = 5$, exponentiate both sides to base $e$:

\[
e^{\ln 2x^3} = e^5 \quad \text{Make both sides a power of } e.
\]

\[
2x^3 = e^5 \quad \text{Use the inverse property.}
\]

The answer is Choice D.

58. To solve $\ln 2x^3 = 5$

\[
2x^3 = e^5 \quad \text{From Question 57.}
\]

\[
x^3 = \frac{1}{2} e^5 \quad \text{Divide both sides by 2.}
\]

\[
x = \sqrt[3]{\frac{e^5}{2}} \quad \text{Take the cubed root of both sides}
\]

You can check by substitution: $\ln 2 \left(\sqrt[3]{\frac{e^5}{2}}\right)^3 = \ln 2 \left(\frac{e^5}{2}\right) = \ln e^5 = 5$. The answer is Choice C.

59. To solve $20 = 3e^x + 5$ first subtract 5 from both sides:

This gives us $15 = 3e^x$. The answer is Choice D.

60. To solve $20 = 3e^x + 5$

\[
3e^x - 15 \quad \text{From Question 59.}
\]

\[
e^x = 5 \quad \text{Divide both sides by 5.}
\]

\[
\ln e^x = \ln 5 \quad \text{Take natural logs of both sides.}
\]

\[
x = \ln 5 \quad \text{Use the inverse property.}
\]

You can check by substitution: $3e^{\ln 5} + 5 = 3 \cdot 5 + 5 = 20$. The answer is Choice E.

61. Use $P(1 + \frac{r}{n})^{nt}$ with $P = 2200$, $r = 0.0382$, and $n = 4$. The balance in year $t$ is $2200(1 + \frac{0.0382}{4})^{4t}$.

Remember that 3.82 per cent is $\frac{3.82}{100} = 0.0382 = 3.82\%$. TIP: To divide 3.82 by 100, move the decimal point of 3.82 two places to the left.

The answer is Choice C.

62. Since you are compounding continuously, use $Pe^{rt}$ with $P = 2200$, $r = 0.0382$. (See previous question.) The balance in year $t$ is $2200e^{0.0382t}$. Note: $2200e^{0.382}$ grows at a continuous rate of 0.382 = 38.2%. Since the balance is none of the choices listed, the answer is Choice E.

TIP: To multiply 0.382 by 100, move the decimal point of 0.382 two places to the right. For example: 0.382 becomes 38.2%.

63. The balance at year $t$ of $1000$ compounded annually at 5% is $1000(1.05)^t$.

a) The amount in year 7, reported to the nearest $0.01$, is $1000(1.05)^7 \approx 1407.10$.

b) To find the total percent by which the account increased at the end of the 7 year period, first find the growth factor $b$, where $1000b = 1407.10$. Divide both sides by 1000.
Thus \( b = 1.40710 \) and \( b = 1 + r \). We have \( r = 0.4017 \), written as a decimal.

The percent growth is \( r = 40.71\% \).

c) The doubling time of the account is the value of \( t \) for which \( 1000(1.05)^t = 2000 \).

Solve \((1.05)^t = 2\)

\[
\ln(1.05)^t = \ln 2 \\
t \ln 1.05 = \ln 2 \\
t = \frac{\ln 2}{\ln 1.05} \approx 14.2 \text{ years}
\]

64. a) Since each value of \( Q \) is halved every 50 years, work backwards to find \( Q_0 \) by doubling 80 so \( Q_0 = 160 \).

b) Complete the next three rows of the table by taking half of the previous row’s output. Check it matches the graph.

c) The half-life is the time it takes to decay by half, or \( 50 \text{ years} \).

d) \( Q = f(t) = 160(0.5)^{t/50} \) (Similar to Question 37c in this packet.

A longer method to get this formula is use the procedure in Questions 25-26 of this packet.)

e) Solve \( 160(0.5)^{t/50} = 1 \) with logarithms or a table or graphically.

Using a table:

<table>
<thead>
<tr>
<th>( t )</th>
<th>( Q = f(t) ), mg</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>160</td>
</tr>
<tr>
<td>50</td>
<td>80</td>
</tr>
<tr>
<td>100</td>
<td>40</td>
</tr>
<tr>
<td>150</td>
<td>20</td>
</tr>
<tr>
<td>200</td>
<td>10</td>
</tr>
<tr>
<td>250</td>
<td>5</td>
</tr>
<tr>
<td>300</td>
<td>2.5</td>
</tr>
</tbody>
</table>

In \( 367 \text{ years} \) it will first be below 1 mg.

Using logarithms (analytically):

\[
160(0.5)^{t/50} = 1 \\
(0.5)^{t/50} = \frac{1}{160} \\
\log(0.5)^{t/50} = \log \frac{1}{160} \\
\frac{t}{50} \log 0.5 = \log \frac{1}{160} \\
t = \frac{50 \log(1/160)}{\log 0.5} \approx 366.0964
\]

At 366 years the amount is above 1 mg.

So in \( 367 \text{ years} \) it will first be below 1 mg.

Graphically: See the graph to the right.

f) Every year the amount decays by 1.4%.

This is similar to Question 37c.

\( Q = 160(0.5)^{t/50} = 160(0.5^{t/50}) \approx 160(0.986)^t \)

Since 96.6% is kept each year, 1.4% is lost each year.
65. The function \( f(x) = \frac{4}{x^2} \) takes any input and returns 4 divided by the square of the input.

We can replace \( x \) by a placeholder, such as an empty box, i.e. \( f(\square) = \frac{4}{(\square)^2} \).

If \( f \) takes the function \( g(x) = \sqrt{x^2 + 4} \) as an input, then we have

\[
\frac{4}{(\sqrt{x^2 + 4})^2} = \frac{4}{x^2 + 4}
\]

This is as simplified as possible. The answer is Choice A.

66. For the function \( f(x) = \frac{\sqrt{x+1}}{2} \) we can replace \( x \) by a placeholder, such as an empty box, i.e.

\[
f(\square) = \frac{\sqrt{\square} + 1}{2}
\]

If \( f \) takes the function \( g(x) = x^2 + 3 \) as an input, then we have

\[
f(\sqrt{\square}) = \frac{\sqrt{x^2 + 3} + 1}{2} = \frac{\sqrt{x^2 + 3} + 1}{2}
\]

This is as simplified as possible. The answer is Choice B.

67. The answer is Choice B. William’s answer is incorrect. His error was in Step 2. It is false to conclude that \( A \cdot B = 1 \Leftrightarrow A = 1 \) or \( B = 1 \).

There are infinitely many ways for a product of two numbers \( A \cdot B = 1 \).

(For example, \( A = \frac{1}{2}, B = 2 \) is one possibility. \( A = \frac{1}{\sqrt{2}}, B = \sqrt{2} \) is another.)

Note: William would have been correct if he had concluded \( A \cdot B = 0 \Leftrightarrow A = 0 \) or \( B = 0 \), which is called the “Zero Factor Property”.

Had William checked his answer, he would have seen if

\[
x = 4 \Rightarrow (4 - 3)(4 + 3) + 6 = (1)(7) + 6 = 13 \neq 7
\]

\[
x = -2 \Rightarrow (-2 - 3)(-2 + 3) + 6 = (-5)(1) + 6 = 1 \neq 7
\]

Had William used a grapher to check, he would have seen there are two solutions but they are not integers. The correct solution can be solved analytically by expanding: \((x - 3)(x + 3) + 6 = 7\)

\[
(x - 3)(x + 3) = 1
\]

\[
x^2 - 9 = 1
\]

\[
x^2 = 10
\]

\[
x = \pm \sqrt{10}
\]

The exact solutions are \( x = -\sqrt{10} \) and \( x = \sqrt{10} \). (Note: \( \pm 3.1622777 \) is the solution approximated to a mere 7 places and is not exact. To 14 places the value of \( \sqrt{10} \approx 3.16227766016838 \), so any decimal representation is approximate.)
68. The graph of \( p(x) \) is a vertical shift of \( f(x) \) down 5 and horizontal shift left 2. The transformation is \( p(x) = f(x + 2) - 5 \)

![Graphs showing transformation](image1)

69. The graph of \( q(x) \) is a vertical compression of \( f(x) \) by a factor of \( \frac{1}{2} \), followed by a vertical shift down 6 and horizontal shift 5 right. The transformation is \( q(x) = 0.5f(x - 5) - 6 \)

![Graphs showing transformation](image2)

70. The graph of \( q(x) \) is a horizontal reflection of \( f(x) \) (or a reflection of \( f(x) \) about the y-axis), followed by a vertical shift down 4. The transformation is \( r(x) = f(-x) - 4 \).

![Graphs showing transformation](image3)