



**Sensitivity Analysis of the MSAE Regression
Problem:
Post-optimality Analysis of the LHS
Coefficients**

Stephen Lewis and John F. Wellington

Sensitivity Analysis of the Left-hand Side Coefficients

Simultaneous Changes

Illustrative Problem

**Minimize $e_1 + e_2 + e_3 + e_4 + e_5 + e_6 + e_7 + e_8 + e_9 + e_{10}$
subject to**

$$b_1 + 0.499b_2 + 11.1b_3 + e_1 = 11.14$$

$$b_1 + 0.558b_2 + 8.9b_3 + e_2 = 12.74$$

$$b_1 + 0.604b_2 + 8.8b_3 + e_3 = 13.13$$

$$b_1 + 0.441b_2 + 8.9b_3 + e_4 = 11.51$$

$$b_1 + 0.550b_2 + 8.8b_3 + e_5 = 12.38$$

$$b_1 + 0.528b_2 + 9.9b_3 + e_6 = 12.60$$

$$b_1 + 0.418b_2 + 10.7b_3 + e_7 = 11.13$$

$$b_1 + 0.480b_2 + 10.5b_3 + e_8 = 11.70$$

$$b_1 + 0.406b_2 + 10.5b_3 + e_9 = 11.02$$

$$b_1 + 0.467b_2 + 10.7b_3 + e_{10} = 11.41$$

b_1, b_2, b_3 unrestricted in sign

$e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}$ unrestricted in sign.



Sensitivity Analysis of the MSAE Regression Problem: Post-optimality Analysis of the LHS Coefficients

Stephen Lewis and John F. Wellington

We can identify the allowable simultaneous variations in the LHS coefficients that preserve the LP solution

Example

$$\text{Constraint 1: } b_1 + 0.499b_2 + 11.1b_3 + e_1 = 11.14$$



Each coefficient may vary simultaneously within identifiable % bounds

Constraint	Left-hand Side (LHS) Coefficients		Allowable Simultaneous % Decrease in LHS Coefficients	Allowable Simultaneous % Increase in LHS Coefficients
1	0.499	11.1	0.805	4.757
2	0.558	8.9	5.345	0.872
3	0.604 ¹	8.8 ¹	-	-
4	0.441	8.9	0.966	5.769
5	0.550	8.8	0.883	5.412
6	0.528	9.9	5.070	0.842
7	0.418	10.7	5.122	0.879
8	0.480 ¹	10.5 ¹	-	-
9	0.406 ¹	10.5 ¹	-	-
10	0.467	10.7	0.844	4.975

¹ LHS coefficients for a binding constraint. Any variation in their values will change the LP solution.



**Sensitivity Analysis of the MSAE Regression Problem:
Post-optimality Analysis of the LHS Coefficients**

Stephen Lewis and John F. Wellington

Overview

- **Linear regression under the minimum sum of absolute errors (MSAE) criterion is used in a wide variety of modeling situations:**

Asset Valuation

**Residential property valuation
Robotic technologies
Financial modeling
Modeling heart data**

- **MSAE regression problem is formulated as linear programming model**
- **In some situations, the left hand side (LHS) coefficients of the LP formulation are subject to errors of collection, recording, or transmission**

We identify the solution's sensitivity to the values of the LHS coefficients

See the interactive illustration