Answers to Practice Questions for Test 3

Review the questions from old homework assignments and quizzes. The Web Page holds a current list.

1. If R = red chip and B = black chip, which integer is modeled by *RBBRRRRRR*? –5

2. Let the letters p, q and r represent different primes. Then p^2qr^3 has 24 different divisors. So would p^{23}. Use p, q and r to describe all whole numbers having exactly the following number of divisors.
   a. numbers with 2 divisors are represented by numbers of the form p.
   b. numbers with 4 divisors are represented by numbers of the form p^3 and pq.

      $4 = 1\times 4$ so we have p^3;
      $4 = 2\times 2$ so we have pq

   c. numbers with 12 divisors are represented by numbers of the form p^{11}, pq^5, pqr^2 and p^2q^3.

      $12 = 1\times 12$ so we have p^{11};
      $12 = 2\times 6$ so we have pq^5;
      $12 = 2\times 2\times 3$ so we have pqr^2;
      $12 = 3\times 4$ so we have p^2q^3;

   d. numbers with 3 divisors are represented by numbers of the form p^2.

3. Determine if the following is true or false without performing the actual division.
   a. 13 | 2600000000052 is true since 13 | 2600000000000 and 13 | 52
   b. 7 | 14,000,000,000,006 is false since 7 | 14,000,000,000,000 and 7 does not divide 6.

4. Use divisibility tests to answer the following:
   a. 1234567890**0987654321** since

```
1  2
3  4
5  6
7  8
9  0
0  9
8  7
6  5
4  3
2  1
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45-45=0
```

   and 0 is divisible by 11

   Another choice is 12345678900123456789.

   b. 12345**1**1**2**2**3** since

```
1     2
3     4
5     __
1     __
2     __
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12-(6+__+__+__)=0
```

   if the sum of the three missing digits is 6.

   Many combinations make six, including 1+2+3 or
   6+0+0 or
   5+1+0 or
   4+2+0 or
   3+3+0 or
   2+2+2 or
   1+1+4

5. Use a calculator to give a prime factorization of 128304. Write your answer as a product of powers of primes. Then determine how many divisors it has.

   $128304 = 2^4 \cdot 8019$ but 8019 is divisible by 3. Therefore

   $128304 = 2^4 \cdot 3^6 \cdot 11$

   From this we see 128304 has $5\cdot 7\cdot 2 = 70$ divisors.

6. Is $2^{500}11^{97}$ a factor of $2^{1001}11^{98}$? Yes, if $k = 2^{501}11$, then $2^{501}11^{97}\cdot k = 2^{1001}11^{98}$.

7. Use the definition of *divides* to show that each of the following is true.
(Hint: Find k that satisfies the definition of *divides*.)
   a. $p^3q^5r \mid p^5q^{13}r^7s^2$ If $k = p^2q^8r^6s^2$ then $p^5q^{13}r^7s^2 = k \cdot p^3q^5r$.
   b. $7 \mid 0$ If $k = 0$ then $0 = 7 \cdot k$.
   c. $0 \mid 0$ For any k then $0 = 0 \cdot k$.

6. How many divisors does each of the following have?
   a. $24 = 2^3 \cdot 3$ has $4 \cdot 2$ or 8 divisors.
   b. $7^{11} \cdot 19^{16} \cdot 23^{17}$ has $12 \cdot 17 \cdot 18$ or 3672 divisors.
   c. $9^4 \cdot 11^3 = 3^2 \cdot 3^2 \cdot 3^2 \cdot 3^2 \cdot 11^3 = 3^8 \cdot 11^3$ has $9 \cdot 4$ or 36 divisors.

9. a. $2^3 \cdot 3^2$
   b. $3^3 \cdot 5^3 \cdot 7^9 \cdot 11^5$

10. If $\text{GCD}(a, b) = 2 \cdot 3$ and $\text{LCM}(a, b) = 2^2 \cdot 3^3 \cdot 5$ and $b = 2^2 \cdot 3 \cdot 5$ then what is a?

$$\text{GCD}(a, b) \cdot \text{LCM}(a, b) = a \cdot b$$
$$(2 \cdot 3)(2^2 \cdot 3^3 \cdot 5) = a \cdot 2^2 \cdot 3 \cdot 5$$
$$2^3 \cdot 3^4 \cdot 5 = a \cdot 2^2 \cdot 3 \cdot 5 \text{ so } a = 2 \cdot 3^3$$