

## Why are the logarithmic properties true?

### Key

1.
  - a. Let  $\log_b Q = x$
  - b. Equation 1a in exponential form:  $b^x = Q$
  - c. Let  $\log_b R = y$
  - d. Equation 1c in exponential form:  $b^y = R$
  - e.  $Q R = b^x b^y$  substituting the results from 1b and 1d.
  - f.  $QR = b^{x+y}$  by properties of exponents.
  - g. Equation 1f in exponential form:  $x + y = \log_b QR$
  - h. Eliminating  $x$  and  $y$  in 1g by substituting 1a and 1c:  
 $\log_b Q + \log_b R = \log_b QR$
  
2.
  - a. Let  $\log_b Q = x$
  - b. Equation 2a in exponential form:  $b^x = Q$
  - c.  $Q^k = (b^x)^k$  if we substitute 2b.
  - d.  $Q^k = b^{xk}$  by laws of exponents
  - e. Equation 2d in exponential form:  $kx = \log_b Q^k$
  - f. Eliminating  $x$  in 2e by substituting 2a:  
 $k \log_b Q = \log_b Q^k$
  
3. 
$$\log_b \left( \frac{U}{V} \right) = \log_b (UV^{-1})$$
$$= \log_b UV^{-1}$$
$$= \log_b U + \log_b V^{-1}$$
$$= \log_b U - \log_b V$$