

Why are the logarithmic properties true?

1. Complete the blanks and boxes to show $\log_b QR = \log_b Q + \log_b R$

- Let $\log_b Q = x$.
- Write the equation in 1a in exponential form: _____
- Let $\log_b R = y$.
- Write the equation in 1c in exponential form: _____
- $QR = b^x \cdot \square$ if we substitute the results from 1b and 1d.
- $QR = b^{\square}$ by properties of exponents. (Hint: See the first rule in the second blue box on page 139 of your text.)
- Now write the equation in 1f in logarithmic form: $(QR) = b^{\square}$ means $\log_b \square = \square$
- Eliminate x and y in the equation in 1g by substituting the equations in 1a and 1c:

2. Complete the blanks and boxes to show $\log_b Q^k = k \cdot \log_b Q$

- Let $\log_b Q = x$.
- Write the equation in 2a in exponential form: _____
- $Q^k = (\square)^k$ if we substitute 2b.
- $Q^k = b^{\square}$ by properties of exponents. (Hint: See third rule in the second blue box on page 139 of your text.)
- Now write the equation in 2d in logarithmic form: $(Q^k) = b^{\square}$ means $\log_b \square = \square$
- Eliminate x in the equation in 2e by substituting the equation in 2a:

3. Complete the boxes to show $\log_b \frac{Q}{R} = \log_b Q - \log_b R$

$$\begin{aligned} \text{Since } \frac{1}{R} = R^{\square}, \text{ we have } \log_b \frac{Q}{R} &= \log_b (Q \cdot \square) \\ &= \log_b (Q \cdot R^{\square}) \end{aligned}$$

$$= \text{_____ using the property in 1h above.}$$

$$= \text{_____ using the property in 2f above.}$$