Practice Questions to Review for the MA 153 Final

Complete the blanks with the information from your instructor:

Date of Final: ____________

Time: ___________________

Room: __________________

The location and time for your section is also at the Web Site
`http://www.ipfw.edu/math/courses/ma153.shtml#final`
There you can find review session opportunities.

<u>Bring the following items to the final</u>:

- ✓ **your graphing calculator**
- ✓ **Number 2 pencils**
- ✓ **ID Number** (You can get this on OASIS after logging into http://my.ipfw.edu. You can also get it from your instructor.)

<u>The final exam will evaluate how well you meet the course goals of MA 153:</u>

- Highlight the link of mathematics to the real world.
- Develop a wide base of mathematical knowledge, including
  - basic skills and concepts,
  - a functional view of mathematics, including graphical, analytical, numerical, and contextual viewpoints (Note: using these four representations is the *Rule of Four*),
  - properties and applications of some of the basic families of functions,
  - geometric visualization,
  - problem solving, predicting, critical thinking, and generalizing.
- Incorporate the use of general academic skills such as
  - communicating mathematics concepts,
  - understanding and using technology.

Just like the chapter exams throughout the semester, this exam tests your ability to interpret detailed, precisely worded directions. Be sure to read the directions carefully and do all that is asked.

<u>Format of the exam:</u> The actual final exam will consist of both multiple-choice questions and open ended (constructed response) questions. Include units in your answers whenever appropriate. You may certainly use your calculator (but not its manual). In fact, some questions will *require* a graphing calculator.

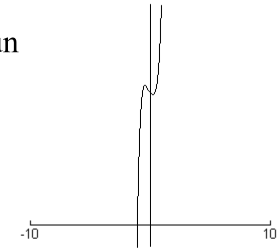
NO formula sheets, notes, books, or other external sources may be used.

<u>For the open ended questions, you will need to show all of your work.</u> If you are basing your reasoning on a graph, then sketch a labeled graph, with numerical values on the axes. If you base your reasoning from a table, you must include the table, which consists of at least five sets of entries. Note: simply restating the question is not explaining your reasoning. For example, if asked to explain your reasoning why you claim a certain phone plan is cheapest, it is insufficient to say something like "Plan A is cheapest because it is the lowest price."

<u>How to prepare for the exam</u>: Some of the practice questions that follow are from previous final exams over this material. Note that the exam will NOT look exactly like these questions, so you should also review previous homework assignments, *eHW*, quizzes, and tests, as well as material worked on during class meetings. Topics from Chapter 9 will receive more of an emphasis than earlier chapters.
Keep the *Rule of Four* in mind when solving problems, just as you have done throughout the semester.

Sample Questions for the Final Exam

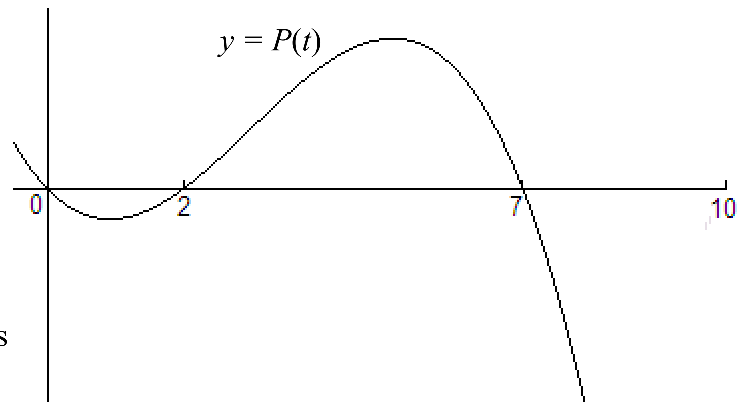
1. Suppose you and Charlie are working together in a group to determine the long run behavior of $f(x) = 60 - 8x + 15x^2 + 25x^3 - 4x^4 + 40x^5 + x^6$. Charlie uses his graphing calculator in the window $-10 \le x \le 10$ and $-10 \le y \le 100$ and sees the graph shown. Charlie concludes that the long run behavior is as follows: As $x \to -\infty$, then $y \to -\infty$; as $x \to \infty$, then $y \to \infty$.. How should you respond?



$-10 \le x \le 10 \quad -10 \le y \le 100$

A. "Good job, Charlie!"
B. "Sorry, Charlie!
As $x \to -\infty$, then $y \to \infty$; as $x \to \infty$, then $y \to \infty$."
C. "Sorry, Charlie!
As $x \to -\infty$, then $y \to -\infty$; as $x \to \infty$, then $y \to -\infty$."
D. "Sorry, Charlie!
As $x \to -\infty$, then $y \to \infty$; as $x \to \infty$, then $y \to -\infty$."
E. "Sorry, Charlie!
As $x \to 0^+$, then $y \to -\infty$; as $x \to 0^+$, then $y \to \infty$."

For **Questions 2-3**, $P(t)$ is a polynomial of degree 3 whose graph is shown.



2. For $0 \le t \le 10$, $P(t)$ describes the temperature of a certain chemical reaction in degrees Celcius, t seconds after the reaction began. Suppose that in the first 2 seconds the minimum temperature that the reaction reaches is –1 degree Celcius and that this temperature is reached 1 second after the reaction began.

Determine the formula for $P(t)$. Then find the minimum temperature that the chemical reaction reaches in the first 10 seconds after it began.

A. $-1^\circ C$ B. $-10^\circ C$ C. $-20^\circ C$ D. $-40^\circ C$ E. None of these

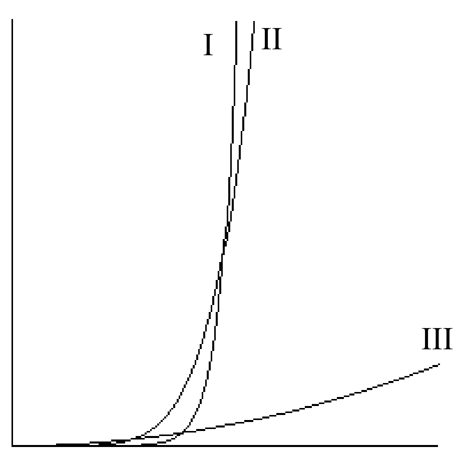
3. A certain power function $Q(t)$ has the same long run behavior as $P(t)$, so much that $Q(t)$ and $P(t)$ look nearly indistinguishable if you graph both of these functions with technology and zoom out for very large values of x. This tells us that it would not be sensible to use $P(t)$ to model the temperature of the reaction for *all* $t \ge 0$. What is the formula for $Q(t)$?

A. $Q(t) = -t^3$ B. $Q(t) = t^3$ C. $Q(t) = -\frac{1}{6}t^3$ D. $Q(t) = \frac{1}{6}t^3$ E. None of these.

4. Global graphs of $y = 1000x^2$, $y = x^6$, and $y = 4^x$ are shown.

Which function corresponds to which?

A. I: $y = 1000x^2$ II: $y = x^6$ III: $y = 4^x$
B. I: $y = x6$ II: $y = 1000x^2$ III: $y = 4^x$
C. I: $y = 4^x$ II: $y = 1000x^2$ III: $y = x^6$
D. I: $y = 4^x$ II: $y = x^6$ III: $y = 1000x^2$
E. None of these



Questions 5-7

The EDI pharmaceutical company has recently acquired the historic Archer building for use as its head office and is gradually moving its employees into this building. The table gives the number, $E(t)$, of EDI employees who have their offices located in the Archer building t months after the initial acquisition of the building.

t	$E(t)$
0	0
1	30
2.02	48
2.83	60

5. The data for $E(t)$ is modeled by a power function.
Find the formula for $E(t)$. Which of the following would be closest to the value of $E(7)$?
A. 100 B. 110 C. 120 D. 130 E. 210

6. Unfortunately, many of the employees of EDI who have their offices located in the Archer building have contracted a mysterious disease which incapacitates them for weeks at a time. The table gives the number, $S(t)$, of EDI employees who have their offices located in the Archer building *and* are sick t months after the initial acquisition of the building.

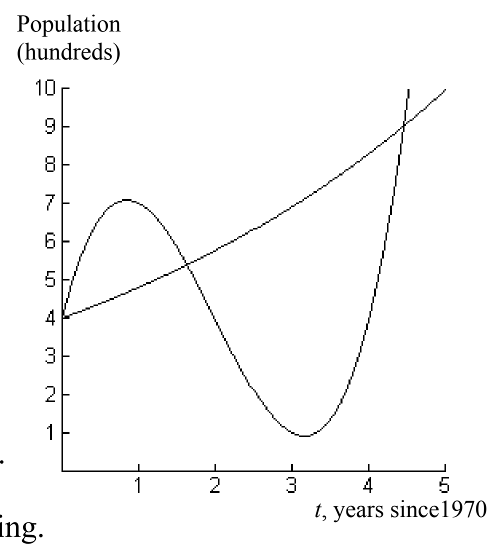
t	$S(t)$
0	0
1.05	6
2.05	15
2.98	25

The data for $S(t)$ is modeled by a power function. Find the formula for $S(t)$.
Which of the following would be closest to the value of $S(11)$?
A. 30 B. 70 C. 110 D. 120 E. 150

7. When the ratio of number of sick employees in a building to total number of employees in a building is greater than 0.75 the building is declared to have sick building syndrome and is closed down for health inspection. How many months after the Archer building is first acquired by EDI will it be closed for health inspection? Select the one closest to your answer.
A. 10 days B. 2 months C. 5 months D. 7 months E. 16 months

8. The population (in hundreds) of the town *Polynomia* grows according to $P(t) = t^3 - 6t^2 + 8t + 4$, where $t = 0$ corresponds to January 1, 1970.

The population (in hundreds) of the town *Exponentia* is given by $E(t)$, where again $t = 0$ corresponds to January 1, 1970. The town initially has 4 hundred people when $t = 0$ and it increases by 20% each year.



The graph shows the populations over the first five years. If the population follows these mathematical models, which of the following must be true? Select the best response.

A. The population of *Polynomia* is always more than 90 people.

B. After 1974 the populations of both towns are always increasing.

C. The population graphs will intersect a total of four times. The population of *Exponentia* will overtake and exceed the population of *Polynomia* sometime after the year 2028.

D. Both A and B are true.

E. All of the above are true.

Questions 9-10

The volume of pollutants (in millions of cubic feet) in a certain reservoir is given by

$$P(t) = 360 + 9t$$

where t is in years. The total volume of the reservoir (which includes both pollutants and water and also in millions of cubic feet) is gradually increasing and is given by

$$R(t) = 12,000 + 12t$$

Let $C(t)$ be the fraction of the reservoir's total volume that consists of pollutants.

Write an expression for $C(t) = \dfrac{P(t)}{R(t)}$ in terms of t.

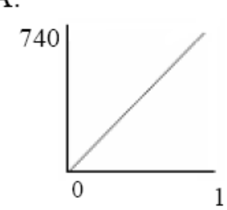
9. In year $t = 0$, what percent of the reservoir's total volume consists of pollutants?
A. 0.3% B. 3% C. $33\frac{1}{3}$ % D. $66\frac{2}{3}$% E. None of these

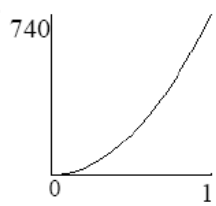
10. According to the mathematical model, if these trends were to continue for many, many years, about what percentage of the reservoir's total volume would eventually consist of pollutants?
A. 0.3% B. 3% C. $33\frac{1}{3}$ % D. $66\frac{2}{3}$% E. None of these

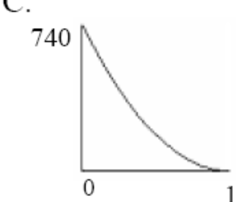
Questions 11-13

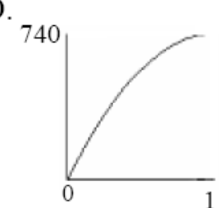
The formula $E = 7.4Lh^2$ is used by marine geologists to find the energy, E (in foot-pounds) delivered by an ocean wave with length L (feet) and height h (feet).

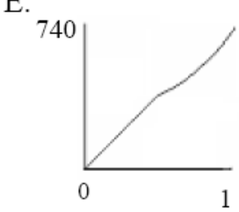
11. Write a function which describes the relationship between E and L for a 10-foot high wave. Sketch a graph of the function $E(L)$.

A.


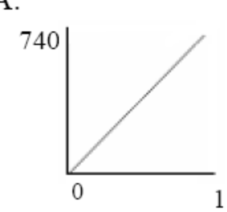
B.


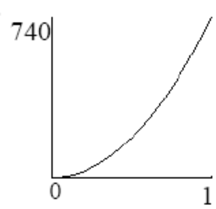
C.


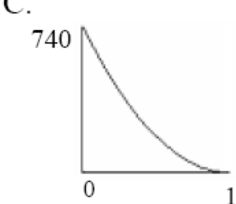
D.


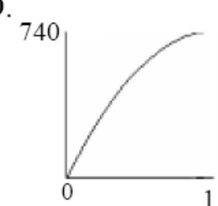
E.


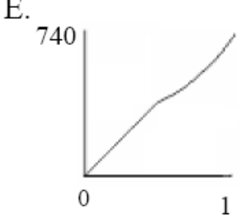
12. Write a function which describes the relationship between E and h for a 100-foot long wave. Sketch a graph of the function $E(h)$.

A.


B.


C.


D.


E.


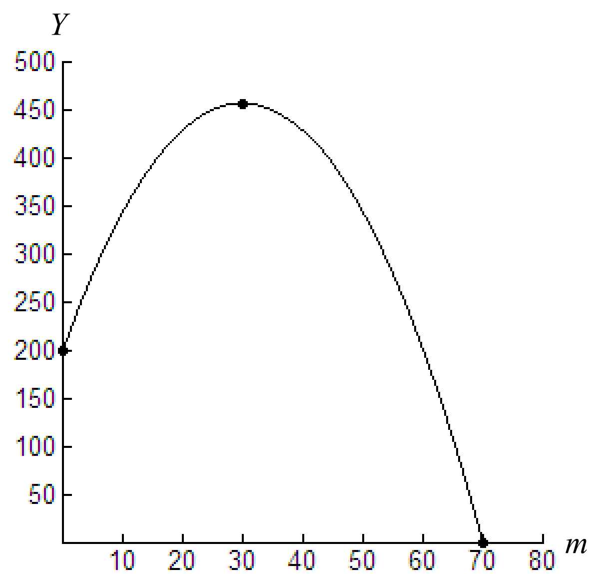
13. Approximate the height of a 100-foot long wave if it delivers 60,000 foot-pounds of energy.
A. 0.81 feet B. 9 feet C. 81 feet D. 86 feet E. None of these.

Questions 14-19

In an apple orchard, the yield, Y, of apples (in bushels) is a function of the amount, m, of fertilizer (in pounds) used on the orchard, so we have $Y = f(m)$. See the graph below.

14. The statement $f(30) = 450$ means
A. The yield ranges from 30 to 450 bushels.
B. When 30 lb of fertilizer is applied, the yield is 450 bushels of apples.
C. For every 30 lb of fertilizer added to the orchard, you increase the yield by 450 bushels.
D. When 450 lb of fertilizer is applied, the yield is 30 bushels of apples.
E. You apply 30 to 450 pounds of fertilizer to the orchard.

15. The vertical intercept for the graph represents:
A. The maximum yield of the orchard.
B. The amount of fertilizer that must be applied to produce a maximum yield.
C. The yield without applying any fertilizer at all.
D. The initial amount of fertilizer applied to the orchard.
E. The amount of fertilizer that will kill all the trees and produce no yield at all.



16. Use the graph of the function to estimate the range.
A. $0 \le f(m) \le 70$
B. $200 \le f(m) \le 450$
C. $70 \le f(m) \le 200$
D. $70 \le f(m) \le 450$
E. $0 \le f(m) \le 450$

17. For what values of m is the function increasing?
A. $200 < m < 450$
B. $0 < m < 450$
C. $0 < m < 30$
D. $30 < m < 70$
E. None of these

18. For what values of m is the function concave up?
A. $200 < m < 450$
B. $0 < m < 450$
C. $0 < m < 30$
D. $0 < m < 70$
E. None of these

19. For what values of m is $Y > 200$?
A. $200 < m < 450$
B. $60 < m < 450$
C. $60 < m < 70$
D. $0 < m < 60$
E. None of these

20. In year $t = 0$, the balance of an account is \$2200. The account earns 3.82% annual interest, compounded quarterly. Find the amount in year t.

A. $2200(1.382)^{4t}$ B. $2200(1+\frac{3.82}{4})^{4t}$ C. $2200(1+\frac{0.0382}{4})^{4t}$

D. $2200(1+\frac{3.82}{4})^{t}$ E. $2200(1+\frac{0.382}{4})^{4t}$

21. In year $t = 0$, the balance of an account is \$2200. The account earns 3.82% annual interest, compounded continuously. Find the amount in year t.

A. $2200e^{1.382t}$ B. $2200e^{1.0382t}$ C. $2200(e\cdot 1.382)^{t}$ D. $2200e^{0.382t}$ E. None of these

22. In the year 2000 the population P of a town was 11,500. It grew by 275 people every year.
In the year 2000 the population Q of a town was 2,000. The town grew by 20% every 5 years.
Find when the population of Q overtakes the population of P.
Select the response which is closest to the answer.

A. 1.97 years B. 10.33 years C. 76.5 years D. 314.6 years E. Q will never overtake P.

Questions 23-24

The amount Q of drug present in a person's body is $Q = 20(0.4)^t$, where Q is in milligrams at time t, and t is in hours.

23. What percent of the drug is lost per hour?

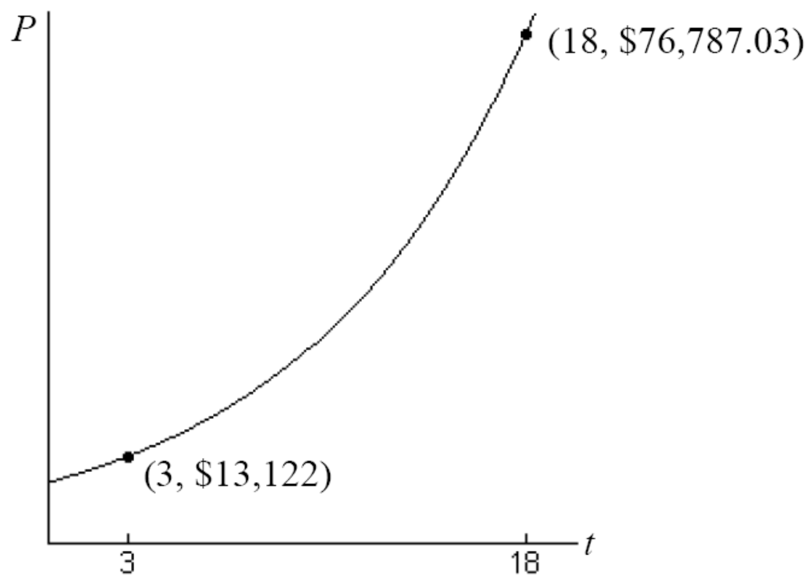
A. 4% B 20% C. 40% D. 6% E. 60% F. 80%

24. What is the growth factor?

A. 0.4 B. 4.0 C. 6.0 D. 20 E. 60 F. 80

Questions 25-26

The graph gives the balance, P, of an investment in year t. Find a possible formula for $P = f(t)$ assuming the balance grows exponentially.



25. Which amount is closest to the initial balance?

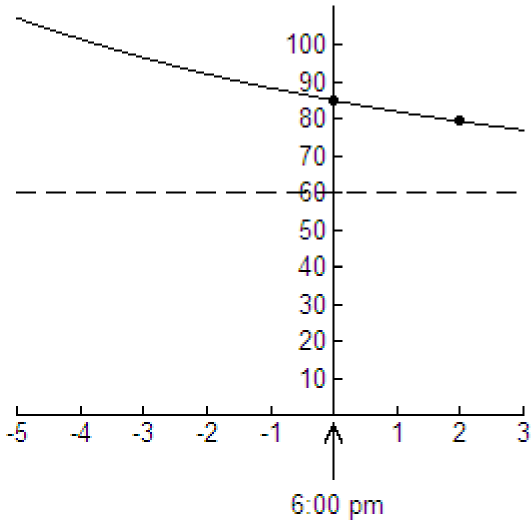
A. \$5,424
B. \$5,454
C. \$9,216
D. \$10,122
E. \$11,664
F. \$41,812

26. What annual interest rate does the account pay?

A. 1.125%
B. 11.25%
C. 12.5%
D. 34%
E. 112.5%
F. 125%

27. You win a contest to have dinner at the mansion of the NCIS* Special Agent Leroy Jethro Gibbs. Unfortunately, when given the pre-dinner tour of the premises, the butler is discovered in the wine cellar, sprawled dead on the floor. Fortunately, medical examiner Dr. Donald "Ducky" Mallard and forensics specialist Abby Sciuto are part of the dinner party, and have brought their probes with them. At 6:00 pm, they use a probe to determine that the body temperature is 85ºF. Two hours later, after dinner, they take another temperature reading to find the body has cooled to 79.36ºF.

(a) Gibbs shares that he keeps his wine cellar at a constant temperature of 60ºF. Ducky notes that, because of this, the butler's body temperature will decay exponentially**. He sketches the graph shown. **Note:** Abby hints that the graph has the model $y = ab^t + 60$. Give the constants a and b. Which is true? Select one:



A. $a = 25$	G. A and D	M. C and D
B. $a = 60$	H. A and E	N. C and E
C. $a = 85$	I. A and F	O. C and F
D. $b \approx -2.82$	J. B and D	P. None of these
E. $b \approx 0.478$	K. B and E	
F. $b \approx 0.88$	L B and F	

(b) Ducky prefers the equation $y = He^{kt} + 60$. Give the constants H and k. Which is true? Select one:

A. $H = 25$	G. A and D	M. C and D
B. $H = 60$	H. A and E	N. C and E
C. $H = 85$	I. A and F	O. C and F
D. $k \approx -1.04$	J. B and D	P. None of these
E. $k \approx -0.74$	K. B and E	
F. $k \approx -0.128$	L B and F	

(c) Gibbs recalls that his niece made a delivery to the wine cellar earlier that day. House records show that she arrived at 2:45 p.m. It is also known that his butler had a reputation of good health - his body temperature was 98.6ºF. Was the butler already dead when the niece delivered the wine? When was the time of death? Explain your reasoning.

*NCIS is Naval Criminal Investigative Service.
**The effect to which Ducky refers is known as *Newton's Law of Cooling*.

28. The monthly charge for a waste collection service is $32 for 100 kilograms of waste and is $48 for 180 kilograms of waste.

(a) Find a linear formula for the cost, C, of waste collection as a function of the number of kilograms of waste, w.

(b) What is the slope of the line found in part **(a)**? Give units and interpret your answer in terms of the cost of waste collection.

(c) What is the vertical intercept of the line found in part **(a)**? Give units and interpret your answer in terms of the cost of waste collection.

29. A research facility on the Isle of Shoals has 900 gallons of fresh water for a two-month period.

(a) There are 4 members of the research team and each is allotted 3 gallons of water per day for cooking and drinking. Find a formula for $f(t)$, the amount of fresh water left on the island after t days has elapsed, assuming that each member of the team uses their total allotment of water each day.

(b) Evaluate and interpret the following expressions

(i) $f(0)$

(ii) $f^{-1}(0)$

30. In a second hand clothing store in Kampala, Uganda, the table* shown is used to exchange U.S. dollars for shillings. The function is linear.

(a) What is the y-intercept of the graph of this function?

(b) A pair of trousers cost 4000 shillings. How much is this in U.S. dollars?

(c) If you have \$4.00 U.S, can you afford a coat which sells for 8500 shillings?

U.S. dollars	Shillings
\$1.00	2200
\$2.50	5500
\$3.00	6600

*Based on a December 2005 Associated Press report.

31. Ruby's Pier, a summer amusement park, charges \$10.00 for admission. An average of 10,000 people visit the park each day it is open. Consultants predict that for each \$1.00 increase in the entrance price, the park would lose an average of 500 daily customers.

(a) Construct a table of values which shows the entrance price, p, and number of tickets sold, N. Your table should have at least five entries.

(b) Let $N = f(p)$. Find a formula for this function.

(c) Add a third column to your table in part **(a)** which gives the daily revenue, R, for each entrance price p. (The *revenue* is the total amount received by the park before any costs are deducted.)

(d) Let $R = g(p)$. Find a formula for this function.

(e) Find the axis intercepts of $N= f(p)$. Interpret what each means to the staff at Ruby's Pier.

(f) Find and interpret the axis intercepts of the revenue function $R = g(p)$.

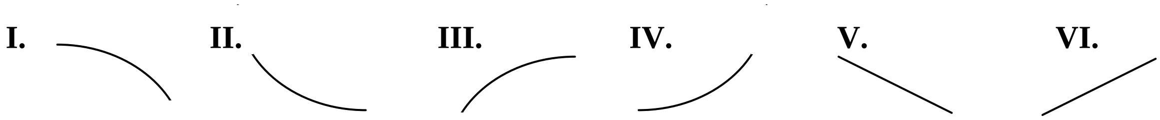
(g) What ticket price maximizes the revenue?

(h) Sketch graphs of $N = f(p)$ and $R = g(p)$ on the same set of axes. Be sure your sketch is properly labeled with the values found in parts **(e)**, **(f)**, and **(g)**.

32. A power function passes through (–5, –40) and (10, 5).

Find a possible formula. Show work to receive any credit

33. For each of the scenarios below, decide which graph (or graphs) are most appropriate.



(a) "Even though the child's temperature is still rising, the penicillin seems to be taking effect."

(b) "Your distance from the Atlantic Ocean, in kilometers per minute, increases at a constant rate."

(c) "The interest on your savings plan is compounded annually.

(d) "At first your balance grows slowly, but its rate of growth continues to increase."

(e) "The annual profit is decreasing at a higher rate each year."

(f) The function has a positive rate of change and the rate of change is decreasing.

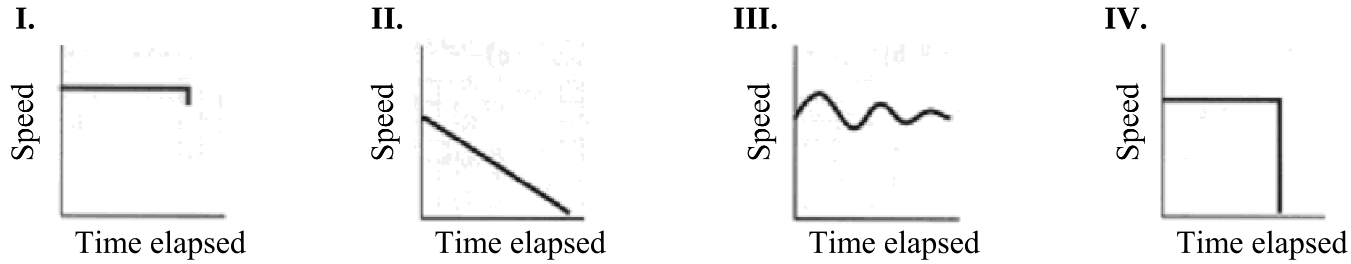
(g) "The price of memory chips isn't decreasing as quickly it used to be."

(h) The function is concave down.

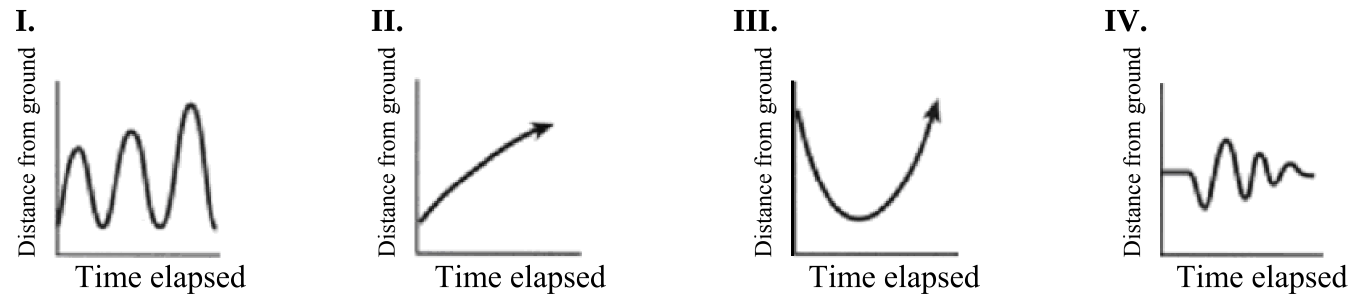
(i) The function is decreasing.

34. Indicate which graph matches the statements. Note the choice of axes.

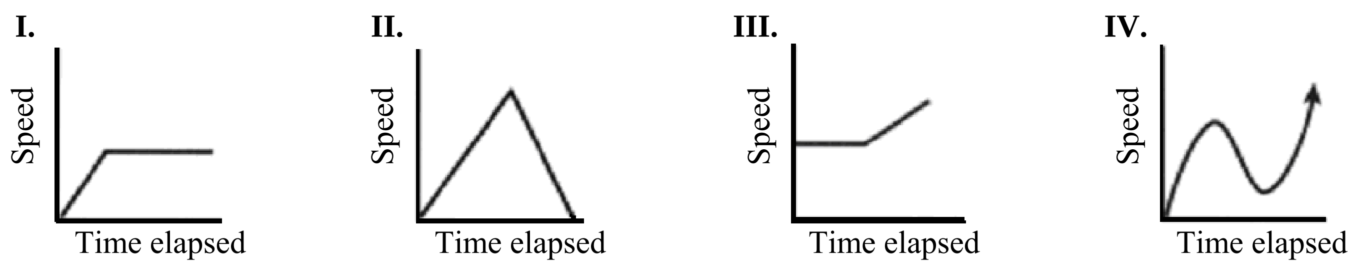
a. A train pulls into a station and lets off passengers.



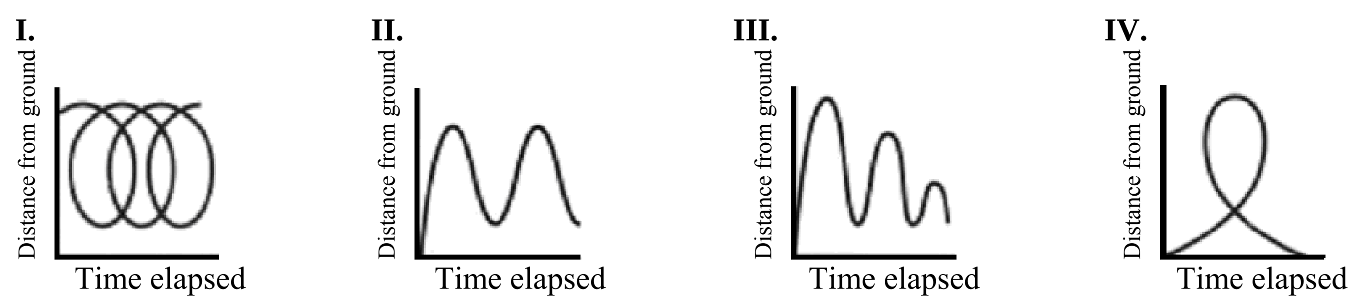
b. A child swings on a swing.



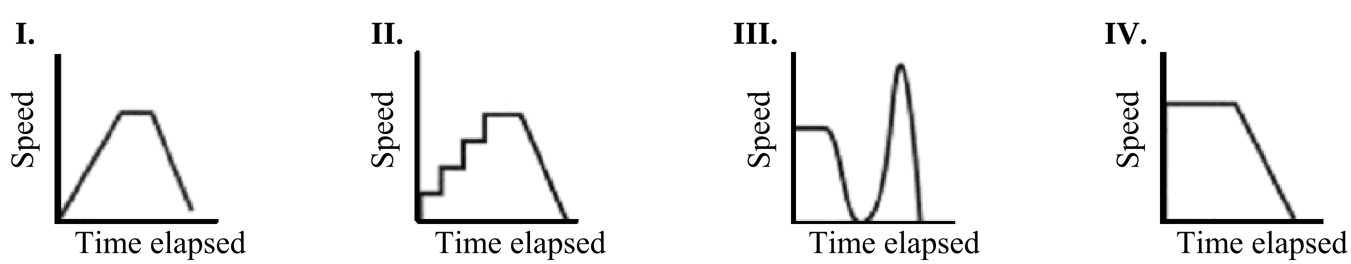
c. A woman climbs a hill at a steady pace and then starts to run down one side.



d. A man takes a ride on a ferris wheel.



e. A child climbs up a slide and then slides down.



35. One description best fits each function. Decide which one, and write its letter in the corresponding blank.

i. _____ $P(t) = 300 - 2t$ ii. _____ $Q(t) = 300e^{0.02t}$

iii. _____ $R(t) = 300(0.98)^t$ iv._____ $S(t) = -\frac{4}{3}t^2 + 300$

A. The population, which began at 300, declines at a constant rate, becoming extinct in 15 years.
B. The population increases exponentially at first and then levels off.
C. The population, which began at 300, is growing at the continuous rate of 2 percent each year.
D. In one year, 98 percent of the population is lost.
E. The population, which was originally at 300, has been increasing at a rate of 98 percent.
F. The population starts at 300 and has dropped to 250 after 25 years.
G. The population, which began at 300, decreases faster and faster.
H. The population, originally at 300, has been decreasing at the annual rate of 2 percent.
I. The population, which was originally at 300, undergoes explosive logarithmic growth, increasing at the annual rate of 2 percent.

36. Determine any x- and y-intercepts and vertical and horizontal asymptotes of each. If none, state so. (Sole use of a graphing calculator may not be the most practical approach for this problem.)

(i) $f(x) = \dfrac{5x^2 - 5}{8000x - 80}$ (ii) $f(x) = \dfrac{5x^2 + 5}{8000x^2 - 80}$ (ii) $f(x) = \dfrac{5x - 5}{8000x^2 + 80}$

37. Short Questions:

(a) Suppose you put $1000 in a bank account at 5% interest compounded continuously. Compute the amount you have at the end of one year, rounded to the nearest cent.

(b) Find log 10. Does log 20 = log 10 + log 10?

(c) Simplify as much as possible: $e^{\ln x^2 + \ln 5}$

(d) If f is a function, describe how the graph of $2f(x + 4) + 3$ is related to that of $f(x)$. Be sure that the order in which the transformations are applied is clear.

(e) Show that $x = 2$ is a solution to the equation $4x + 8 = 4^x$. Then find all values of x which solve $4x + 8 > 4^x$. (Report your answer accurate to 3 decimal places.)

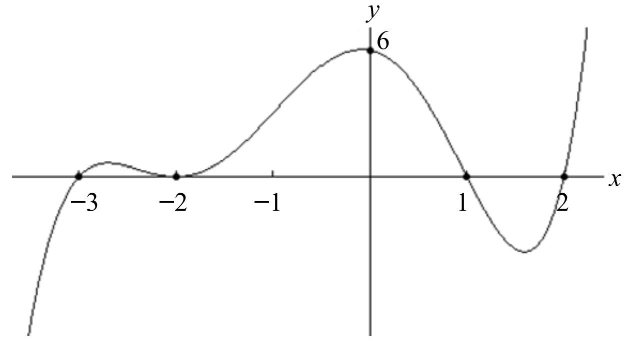
38. Which of the following functions has its domain identical with its range?

A. $f(x) = x^2$ B. $g(x) = \log x$ C. $h(x) = x^3$ D. $i(x) = |x|$ E. None of these.

39. For the graphs in this problem assume all global behavior is shown.

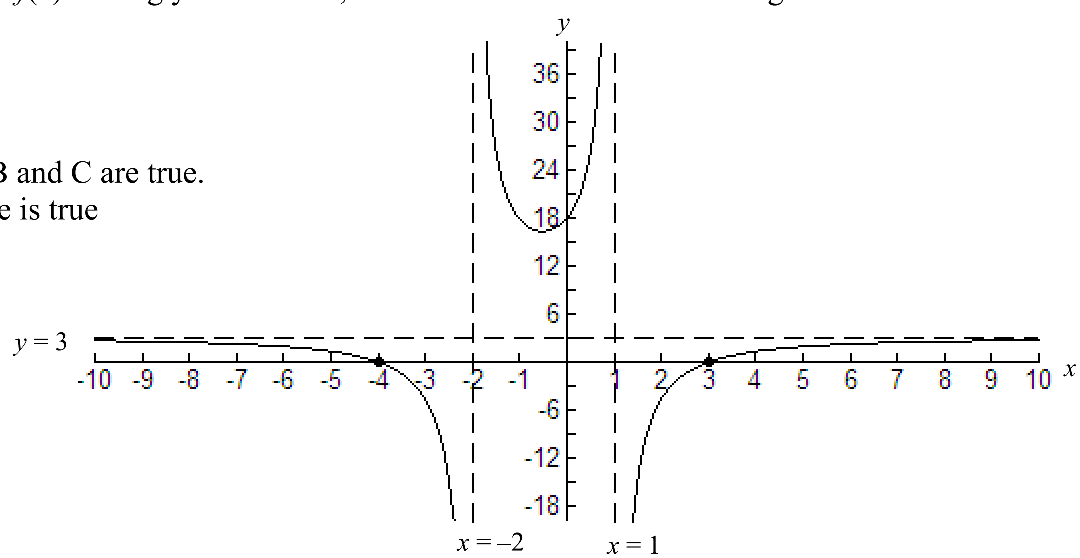
a. Find a possible formula of least possible degree for the function $y = f(x)$. Then use your formula to find $f(3)$.

A. 30
B. 75
C. 306
D. 501
E. None of these



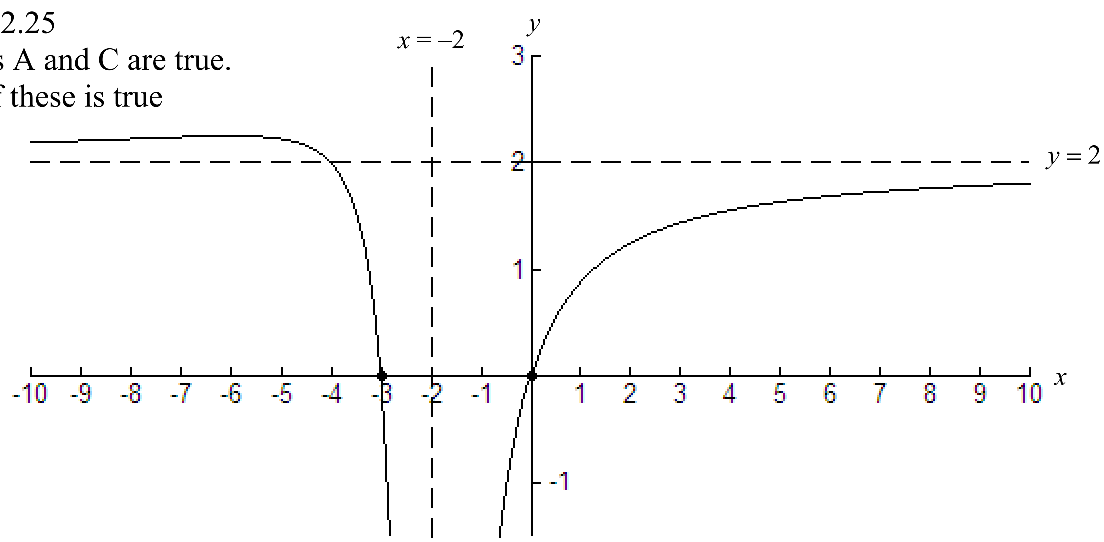
b. Find the formula for $f(x)$. Using your formula, determine which of the following must be true.

A. $f(-1) = f(0)$
B. $f(-7) = f(6)$
C. $f(-3) = f(2)$
D. Choices A, B and C are true.
E. None of these is true

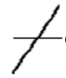
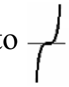
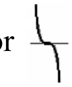
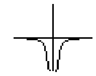
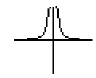


c. Find the formula for $f(x)$. Using your formula, determine which of the following must be true.

A. $f(-1) = -4$
B. $f(1) = 1$
C. $f(-6) = 2.25$
D. Choices A and C are true.
E. None of these is true



40. A rational function $y = f(x)$ has the following properties:

- there is only one zero at 4,
- the short run behavior near that zero looks like  or  (as opposed to  or )
- there is one vertical asymptote at $x = 2$,
- the short run behavior near the vertical asymptote looks like  or 
- the degree of the denominator is the lowest degree possible,
- there is a horizontal asymptote of $y = 0$, and
- $f(0) = -8$

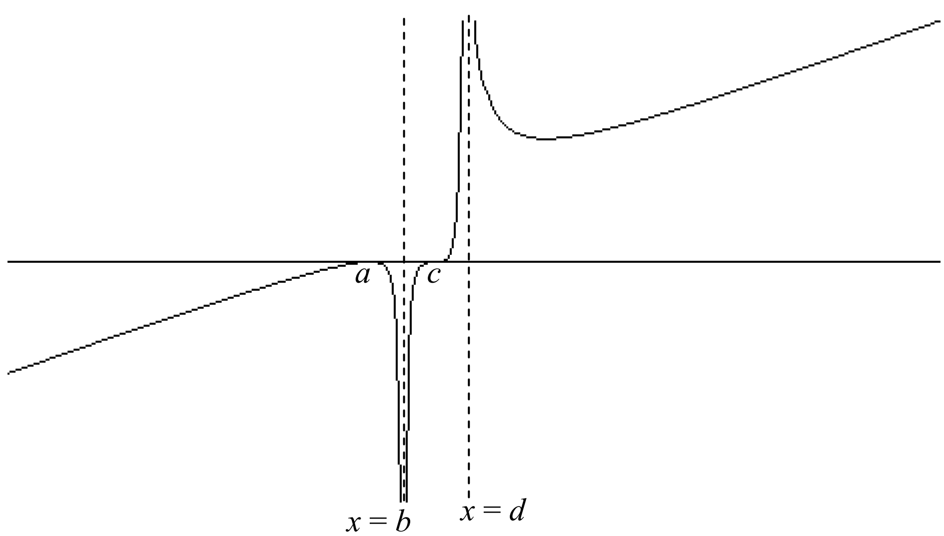
Find the formula for $f(x)$. Then use your formula to find $f(3)$.

A. −16
B. −8
C. −4
D. 8
E. 16

41. Assume *a, b, c*, and *d* are positive real numbers.
The rational function $f(x)$ graphed below has the following properties:

- <u>short run behavior</u>:
  - o zeros are at a, c
  - o vertical asymptotes are at $x = b$ and $x = d$
- <u>long run behavior</u>:
  as $x \to -\infty, y \to -\infty$
  as $x \to \infty, y \to \infty$
  Consequently, there is no horizontal asymptote.

Assume k is some positive real number. Which could be its equation?



A. $f(x) = \dfrac{k(x-a)^2(x-c)^3}{(x-b)^2(x-d)^4}$

B. $f(x) = \dfrac{k(x-a)^2(x-c)^3}{(x-b)^2(x-d)^2}$

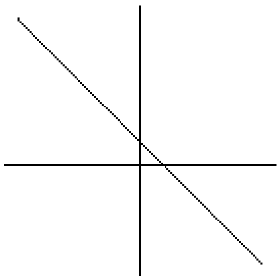
C. $f(x) = \dfrac{k(x-a)^2(x-c)^3}{(x-b)^2(x-d)}$

D. $f(x) = \dfrac{k(x-a)^4(x-c)^3}{(x-b)^3(x-d)^4}$

E. $f(x) = \dfrac{k(x-a)^2(x-c)^3}{(x-b)(x-d)^3}$

42. For each of the graphs below, select the formula beneath the graph which best fits the behavior of the graph. In each case, assume that A, B, and C are positive real numbers. (Circle your choice.)

(I)



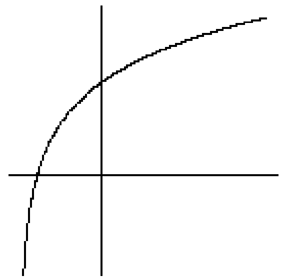
(a) $y = Ax + B$

(b) $y = -Ax - B$

(c) $y = B - Ax$

(d) $y = \dfrac{x+A}{x+A}$

(II)



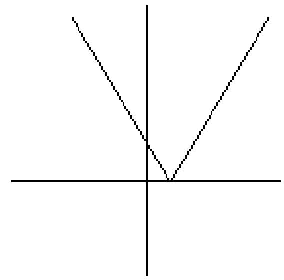
(a) $y = e^{-Ax}$

(b) $y = \log(x - A)$

(c) $y = \log(x + A)$

(d) $y = A^{(x+B)}$

(III)



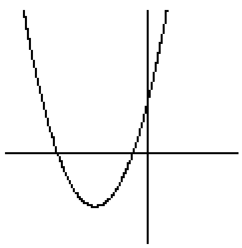
(a) $y = |x - A|$

(b) $y = |x + A|$

(c) $y = |x| - A$

(d) $y = |x| + A$

(IV)



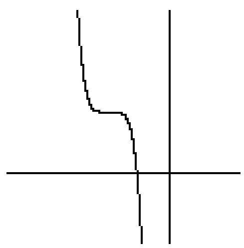
(a) $y = Ax^2 - B$

(b) $y = C - A(x+B)^2$

(c) $y = A(x+B)^2 - C$

(d) $y = A(x-B)^2 - C$

(V)



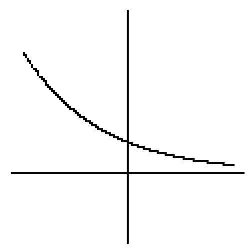
(a) $y = -Ax^5 + B$

(b) $y = Ax^3 + B$

(c) $y = -A(x+B)^5 + C$

(d) $y = A(x+B)^5 + C$

(VI)



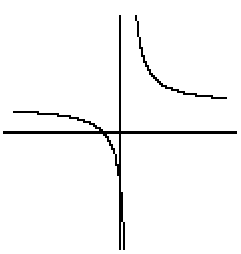
(a) $y = -\ln(x + A)$

(b) $y = -(1/A)^x$

(c) $y = -A^x$

(d) $y = (1/A)^x$

(VII)



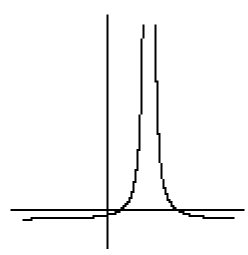
(a) $y = \dfrac{A(x-B)}{x+C}$

(b) $y = -\dfrac{A(x-B)}{x+C}$

(c) $y = \dfrac{A(x+B)}{x-C}$

(d) $y = \dfrac{-A(x+B)}{x-C}$

(VIII)



(a) $y = \dfrac{A}{(x-B)^2} - C$

(b) $y = \dfrac{A}{(x+B^2)} - C$

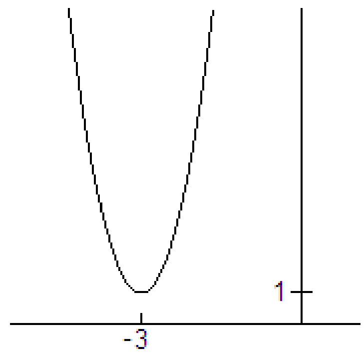
(c) $y = \dfrac{A}{(x-B)} - C$

(d) $y = \dfrac{-A}{(x-B)} - C$

43. A line with slope $-\frac{2}{3}$ passes through the point (60, 30). Find the <u>x-intercept</u> of the line.

A. 105 B. 70 C. 60 D. 30 E. None of these

44. The graph of the function is a translation of $y = 5x^2$, shifted left 3 and up 1. What is the **range** of the graph?



A. all real numbers
B. $y \geq 1$
C. $y \geq -3$
D. $y \geq -1$
E. $y \leq 1$

45. Assuming x, y, and w are positive real numbers, which of the following is $\log \frac{x^3 y^2}{\sqrt{w}}$?

A. $x^3 + y^2 - \sqrt{w}$ B. $\frac{1}{3}\log x + \frac{1}{2}\log y - 2\log w$ C. $3\log x + 2\log y - \frac{1}{2}\log w$

D. $\dfrac{3\log x + 2\log y}{\frac{1}{2}\log w}$ E. None of these

46. Solve for x to the nearest hundredth: $25^x = 3^{600}$
(Most calculators are unable to solve this numerically or graphically due to overflow problems.)

A. 409.56 B. 530.44 C. 204.78 D. No solution E. None of these

47. Find the vertex of the parabola: $y = 4x^2 + 8x + 100$.

A. (−2, 100) B. (1, 130) C. (0, 100) D. (−1, 96) E. None of these

48. Find all the zeros of the polynomial function: $f(x) = 7(x^3 - 3x^2 - 4x)$.

A. −1, 4 B. −4, 1 C. −1, 0, 4 D. −1, 0, 4, 7 E. None of these

49. Find all possible values of x for which $9x^2(x+6)(x-6)^2 \geq 0$.
Support your reason graphically.

A. $-6 \leq x \leq 6$ B. $-6 \leq x \leq 0$ or $x \geq 6$ C. $x \geq -6$ D. $x \leq 6$ E. None of these

50. An initial deposit of $4000 is made in a savings account for which the interest is compounded continuously. If the interest rate is 7.3%, how long will it take, to the nearest 0.01 year, for the investment to triple? Use $A = Pe^{rt}$.

A. 0.15 years B. 2.79 years C. 6.54 years D. 15.05 years E. None of these

51. Given $f(x) = \frac{1}{x^2}$ and $g(x) = \sqrt{x^2 + 4}$, find $f(g(x))$.

A. $f(g(x)) = \dfrac{1}{x^2 + 4}$ B. $f(g(x)) = \dfrac{1}{\sqrt{x^2 + 4}}$ C. $f(g(x)) = x^2 + 4$ D. $f(g(x)) = \dfrac{1}{x^2\sqrt{x^2 + 4}}$

52. Which of the following is true about the graph of $y = f(x) = b^x$? List **all** correct answers.

I. It increases if $b > 1$
II. It decreases if $b < 0$
III. It has y-intercept (0, 1) if $b > 0$.

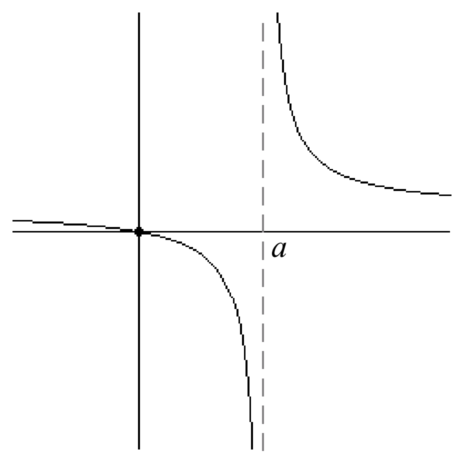
A. I, II and III B. I and II C. II and III D. I and III E. III only.

53. Use what you know about transformations and the graph of $y = \log x$ to answer the following about the graph of $f(x) = 2 + \log(x - 1)$. Which are true? List **all** correct answers.
The graph of $f(x) = 2 + \log(x - 1)$

I. increases for all values of x in its domain.
II. crosses the x-axis at 1
III. never touches the y-axis
IV. passes through the point (2, 2).

Note: Don't be misled by technology when answering this question.
A. I, II and III B. I and II C. II and IV D. I and IV E. I, III and IV only.

54. A function passes through the origin and has a vertical asymptote at $x = a$, where $a > 0$. It has the graph shown. Which could be its equation?



A. $f(x) = \dfrac{1}{x-a}$ B. $f(x) = \dfrac{1}{x+a}$ C. $f(x) = \dfrac{x}{x-a}$

D. $f(x) = \dfrac{x}{x+a}$ E. $f(x) = \dfrac{x}{(x-a)^2}$

55. Let a be some constant. Which is true about $f(x) = \dfrac{2ax}{(x-a)^2}$?

A. Its horizontal asymptote is $y = 2a$. D. Its horizontal asymptote is $y = 0$.
B. Its horizontal asymptote is $y = 2$. E. It has no horizontal asymptote.
C. Its horizontal asymptote is $y = \dfrac{2a}{x}$.

56. The relationship of pH to the hydrogen ion concentration ($[H^+]$) is pH = $-\log [H^+]$.
If the pH is 2.1, what is the hydrogen ion concentration?
A. 0.74 B. 0.008 C. 125.89 D. −0.322 E. −125.89

Questions 57-58:

57. Which of the following is an acceptable first step to solve the equation $\ln 2x^3 = 5$?

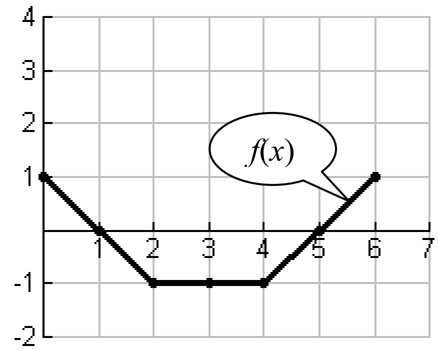
A. $3\ln 2x = 5$ B. $2x^3 = e \cdot 5$ C. $2x^3 = \dfrac{5}{\ln}$ D. $2x^3 = e^5$ E. $\ln 2x^3 = \ln 5$

58. What is the correct **exact** solution to the equation $\ln 2x^3 = 5$?

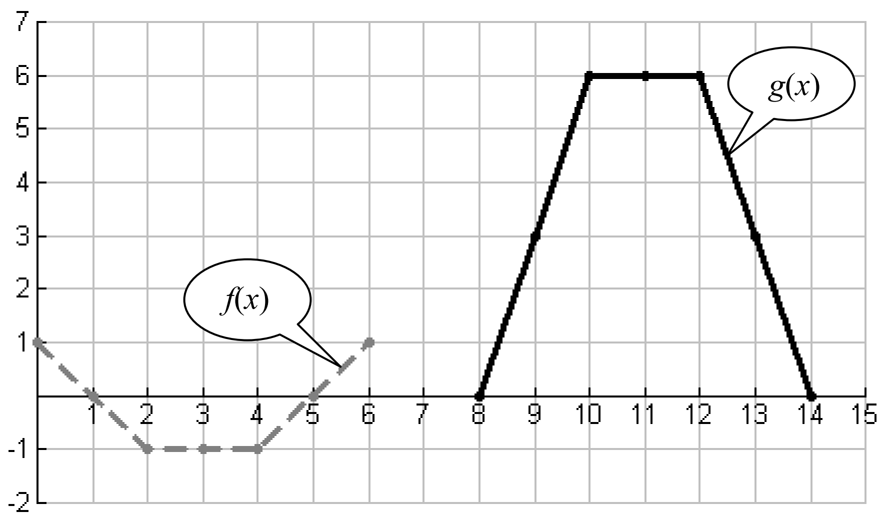
A. $\sqrt[3]{\dfrac{5}{2\ln}}$ B. $\sqrt[3]{\dfrac{5e}{2}}$ C. $\sqrt[3]{\dfrac{e^5}{2}}$ D. $\sqrt[3]{\dfrac{5}{2}}$ E. $\frac{1}{2}e^{5/3}$

Questions 59-60:

The graph of $y = f(x)$ is shown.



59. The function $y = g(x)$ shown below is a transformation of $f(x)$.
Write a rule for $g(x)$ in terms of $f(x)$.



A. $g(x) = -3f(x-8)$

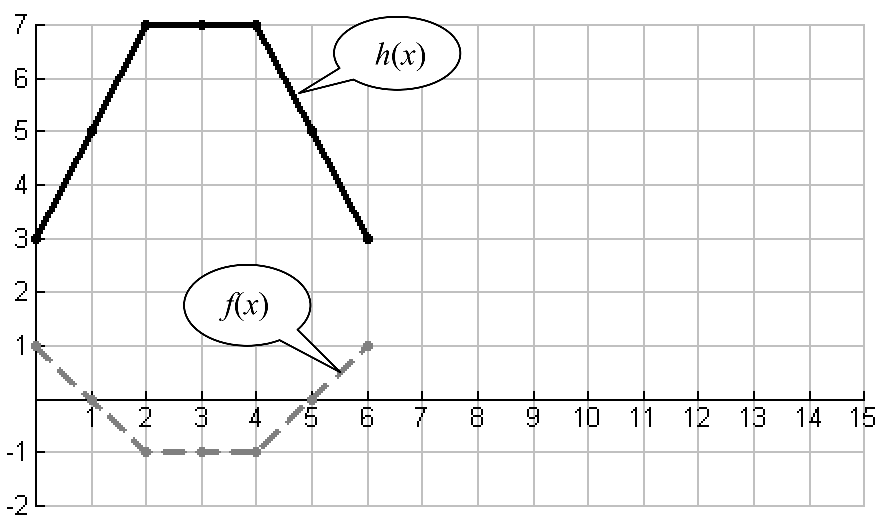
B. $g(x) = -3f(x+8)$

C. $g(x) = -6f(x-8)$

D. $g(x) = -6f(x+8)$

E. $g(x) = -3f(x-8)+3$

60. The function $y = h(x)$ shown below is a transformation of $f(x)$.
Write a rule for $h(x)$ in terms of $f(x)$.



A. $h(x) = -2f(x)+6$

B. $h(x) = -4f(x)+4$

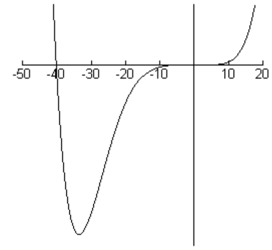
C. $h(x) = -2f(x)+5$

D. $h(x) = -4f(x)+3$

E. $h(x) = -2f(x)+4$

Solutions to Review for Final

1. For positive or negative large values of x,
$f(x) = 60 - 8x + 15x^2 + 25x^3 - 4x^4 + 40x^5 + x^6$ look like
the power function y = x6. We can describe its long run behavior as follows:
As $x \to -\infty$, then $y \to \infty$; as $x \to \infty$, then $y \to \infty$.
Enlarge the viewing window to see that eventually the graph turns around.
Choice **B**.

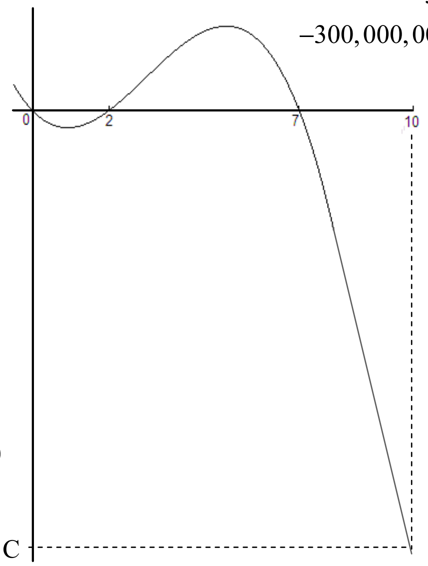


$-50 \le x \le 20$
$-300,000,000 \le y \le 100,000,000$

2. There are zeros at 0, 2, and 7. Therefore:

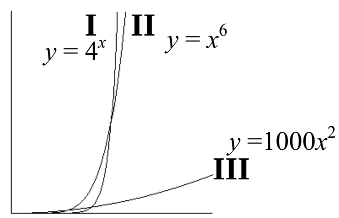
$y = kt(t-2)(t-7)$ passes through $(1, \ -1)$:
$-1 = k(1)(1-2)(1-7)$
$-1 = k(-1)(-6)$
$k = -\frac{1}{6}$

The minimum value of $P(t)$ in the first ten seconds must be $P(10) = -40^\circ\text{C}$.
This can be found using a graph or table or by evaluating $P(t) = -\frac{1}{6}t(t-2)(t-7)$ for $t = 10$.
$P(10) = -\frac{1}{6}(10)(10-2)(10-7) = -\frac{1}{6}(10)(8)(3) = -40$
Choice **D**.



3. $Q(t) = -\frac{1}{6}t^3$ since
$P(t) = -\frac{1}{6}t(t-2)(t-7) = -\frac{1}{6}t^3 + \textit{remaining terms of lower degree}$ (See Section **9.2.**)
Therefore, $P(t)$ and $Q(t)$ look very much alike for large values of t. (Note that the $-\frac{1}{6}$ is not optional.)

4. Since all global behavior is shown, notice in the long run that graph **I** is above graph **II** which is above graph **III**.
Exponential functions eventually outpace power functions, so the graph of $y = 4^x$ will be above graphs of $y = 1000x^2$ and $y = x^6$.
This means that **I** must be the graph of $y = 4^x$.
Power functions with greater degree will outpace those of lower degree, so the graph of $y = x^6$ must be above the graph of $y = 1000x^2$ after some point. (See Section **9.6.**)
Therefore **II** must be the graph of $y = x^6$ and **III** must be the graph of $y = 1000x^2$.
Choice **D**.



5. $E(t) = 30t^{0.668}$. To find $y = kt^p$, notice $E(1) = 30$ so if $t = 1$, then $y = 30$.

Therefore we have $k = 30$, since $30 = k(1)^p = k(1) = k$.

Now use another point to find p for $y = 30t^p$. We used (2.02, 48).

$48 = 30(2.02)^p$

$\frac{48}{30} = (2.02)^p$

$1.6 = (2.02)^p$ So $p = \frac{\ln 1.6}{\ln 2.02} \approx 0.668$.

This means $E(t)=30x^{0.67}$ and $E(7)=30(7)^{0.67}\approx 110$. Choice **B**.

6. $S(t)=5.61x^{1.37}$. To find $y = kt^p$, use two points. We used (2.05, 15) and (2.98, 25).

$$\frac{25}{15}=\frac{k(2.98)^p}{k(2.05)^p}$$

$$\frac{5}{3}=(\tfrac{2.98}{2.05})^p$$

$$p=\frac{\ln(5/3)}{\ln(2.98/2.05)}\approx 1.3655$$

$y=kt^{1.366}$ Now use any other point to find k. We used (1.05, 6)

$6=k(1.05)^{1.366}$

$k\approx 5.61$

$S(t)=5.61x^{1.37}$ Choice **E**.

7. $E(t)=30x^{0.67}$

$S(t)=5.61x^{1.37}$

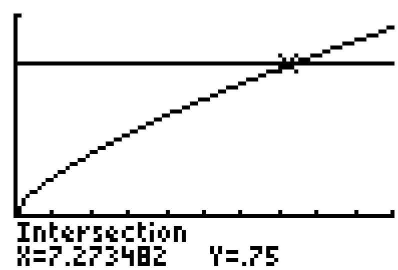
$$\frac{S}{E}=\frac{5.61x^{1.37}}{30x^{0.67}}=0.187x^{0.7}$$

Solve $0.187x^{0.7}>0.75$ by graphing $y = 0.187x^{0.7}$ and the target line $y=0.75$

Perform an INTERSECTION routine or solve $0.187x^{0.7}=0.75$ to find the first time after which the ratio $\frac{S}{E}$ is above 0.75. This is about 7.3 months. Choice **D**.

```
Y1=.187X^.7
Y2=.75
```

Window: $0\le x\le 10,\ -0.25\le y\le 1$



Note: You could also just enter

```
Y1=5.61X^1.37/(30X^.67)
Y2=.75
```

in a grapher, but the parentheses are crucial on a TI-83 or TI-83 Plus.

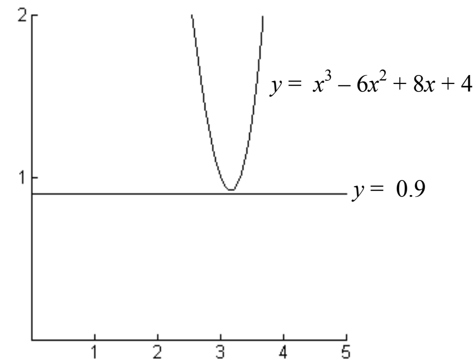
For example, you would NOT get the same function if you just typed

```
Y1=5.61X^1.37/30X^.67
Y2=.75
```

That would give you $y=\frac{5.61x^{1.37}}{30}\cdot x^{0.67}$

which is not what you want at all.

8. The town of *Polynomia* always exceeds 90 people.
A population of 90 people = 0.9 hundred.
Use a graphing calculator to sketch
$y = x^3 - 6x^2 + 8x + 4$ and the line $y = 0.9$
in a viewing window such as
$0 \le x \le 5$ and $0 \le y \le 2$
The polynomial never falls below the line.
You could also use a calculator routine to determine the minimum value of the polynomial in this window, which is (3.1547, 0.920799).



For $t > 0$ the lowest value of this town's population is a mere 92 people!

$P(t) = t^3 - 6t^2 + 8t + 4$ has the same long run behavior as $y = x^3$, so it will continually increase after $t = 4$ or after 1974. Since the town of *Exponentia* begins initially with 400 people and grows by 20% each year, the formula for $E(t) = 4(1.2)^t$. The graph of this function increases for all t. Exponential functions will eventually outpace polynomial functions (see Section **9.6**), so the graphs must cross more than three times. Using graphing technology, we can find that $E(t)$ will intersect $P(t)$ again about 58.88 years after 1970, or in the year 2028. To check this, sketch the difference function $D(t) = E(t) - P(t)$ on a grapher and find when it is zero.

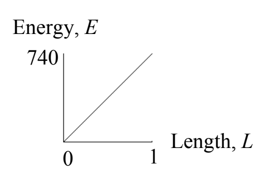
The correct response is Choice **E**, all of the above are true.

9. We have $C(t) = \dfrac{P(t)}{R(t)} = \dfrac{360 + 9t}{12{,}000 + 12t}$. Therefore $C(0) = \dfrac{360 + 9(0)}{12{,}000 + 12(0)} = \dfrac{360}{12{,}000} = 0.03$ or 3%.

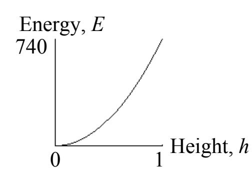
Choice **B**.

10. As t gets larger and larger, the function $C(t) = \dfrac{360 + 9t}{12{,}000 + 12t}$ approaches the ratio of the leading terms, namely $\dfrac{9t}{12t} = 0.75$. Eventually 75% of the reservoir's total volume would consist of pollutants. This can be confirmed with a graph of the function or a view of its table for large values of t. Choice **E**.

11. If $h = 10$, then $E(L) = 7.4(L)(10)^2 = 740L$
The graph is a line through the origin with slope 740.
It passes through the point (1, 740). Choice **A**.



12. If $L = 100$, then $E(h) = 7.4(100)h^2 = 740h^2$
The graph is a parabola ⟶
through the origin also passing through (1, 740). Choice **B**.



13. Find h if $L = 100$ and $E = 60{,}000$ ft-lb.

$$740h^2 = 60{,}000$$

$$h^2 = \tfrac{60{,}000}{740}$$

$$h = \sqrt{\tfrac{6000}{74}} \approx 9.005$$ The wave would be about 9 ft. tall. Choice **B**.

14. 30 lb of fertilizer produces a maximum yield of 450 bushels of apples. Choice **B**.

15. Without applying any fertilizer at all, we see from the graph that the orchard will produce 200 bushels of apples. Choice **C**.

16. The range is $0 \le f(m) \le 450$. Note: You can also write [0, 450]. Choice **E**.

17. The function $f(m)$ is **increasing** for $0 < m < 30$. Choice **C**.
Note: The function f(m) is **decreasing** for $30 < m < 70$.

18. The function $f(m)$ is never **concave up** and is **concave down** for $0 < m < 70$. Choice **E**.

19. $f(m) > 200$ for $0 < m < 60$.
Determine where the graph of $y = f(m)$
is above the line $y = 200$.
The yield is more than 200 bushels of apples
when the amount of fertilizer applied is
more than 0 lb and less than 60 lb.
Choice **D**.

20. Choice **C**.

21. Choice **E**. It should be $2200e^{0.0382t}$

22. $P = 11500 + 275t$ and $Q = 2000(1.2)^{t/5}$
Set the equations equal to each other and solve using
technology. They intersect at $t = 76.5$ years. Choice **C**.

23. $Q = 20(0.4)^t = 20(1 - 0.6)^t$, so 60% of the drug is lost per hour. Choice **E**.

24. Choice **A**.

25. The equation is $P = 9216(1.125)^t$. The initial amount when $t = 0$ is \$9,216. Choice **C**.

26. Since the equation is $P = 9216(1.125)^t = 9216(1+ \underline{\mathbf{0.125}})^t$, the growth rate is 12.5%. Choice **C**.

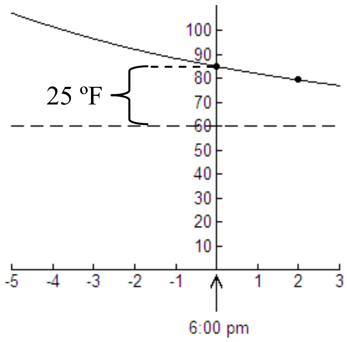
27. (a) We have been given that the equation is of the form $y = ab^t + 60$ and we must find a and b.
When $t = 0$, $y = 85$ ºF:

$85 = ab^0 + 60$

$85 = a + 60$

$a = 85 - 60 = 25.$

Note this is the initial temperature difference between the butler and the room temperature.



We have $y = 25b^t + 60$ and need b.
When $t = 2$, $y = 79.36$ ºF:

$79.36 = 25b^2 + 60$

$19.36 = 25b^2$

$$b^2 = \frac{19.36}{25} = 0.7744$$

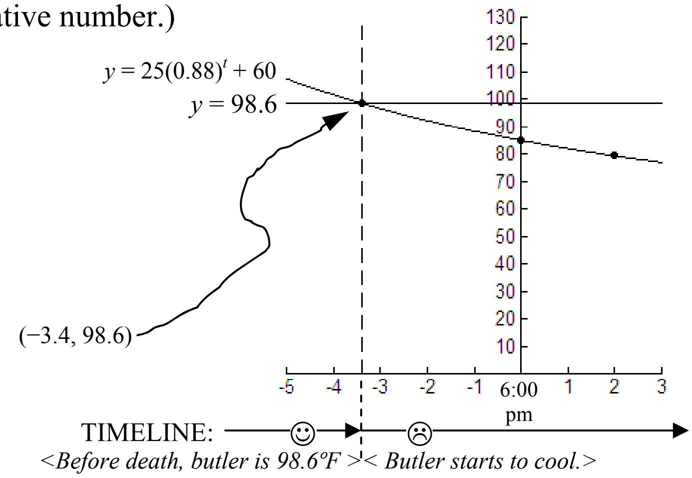
$$b = \sqrt{0.7744} = 0.88$$

The model is $y = 25(0.88)^t + 60$. Check with a grapher or resubstitute the points. Choice **I.**

(b) We must write $y = 25(0.88)^t + 60$ as $y = He^{kt} + 60$.
We can simplify this to writing $25(0.88)^t$ as He^{kt} for some constants H and k.
The constant H is 25.
To find k, set $e^k = 0.88$
so $k = \ln e^k = \ln(0.88)$
To 3 decimal places, $k = \ln(0.88) = -0.128$ and we have $y = 25e^{-0.128t} + 60$.
Again we can check with a grapher or resubstitute the points. Choice **I.**

(c) When his body temperature, y, is 98.6ºF, we will assume the butler was alive.
Set $y = 25(0.88)^t + 60$ and $y = 98.6$ equal to each other to find the time of death.
(From the graph, we expect a negative number.)
Algebraic solution:

$$25(0.88)^t + 60 = 98.6$$
$$25(0.88)^t = 38.6$$
$$(0.88)^t = \tfrac{38.6}{25}$$
$$\ln(0.88)^t = \ln \tfrac{38.6}{25}$$
$$t\ln(0.88) = \ln(38.6/25)$$
$$t = \tfrac{\ln(38.6/25)}{\ln(0.88)} \approx -3.4$$



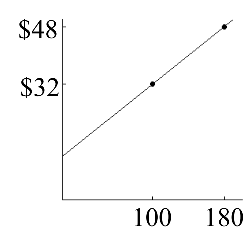
Notice the timeline on the graph:
He died 3.4 hours before 6:00 pm.
This is 3 hours and $0.4 \cdot 60 = 24$ minutes before 6:00 pm. (or 24 minutes prior to 3:00 pm) which is 2:36 pm. Since the house records indicate that the niece arrived at 2:45 pm., the butler was already dead when she arrived.

Note: You could have also have used the equation involving e as shown below.
Since $\ln(0.88) = -0.128$, you reach the same answer:

$$25e^{-0.128t} + 60 = 98.6$$
$$25e^{-0.128t} = 38.6$$
$$e^{-0.128t} = \tfrac{38.6}{25}$$
$$\ln e^{-0.128t} = \ln \tfrac{38.6}{25}$$
$$-0.128t = \ln(38.6/25)$$
$$t = \tfrac{\ln(38.6/25)}{-0.128} \approx -3.4$$

28. (a) It might be helpful to plot the points and organize the information in a table.

w	C
100	\$32
180	\$48



The slope is positive:

$$m = \frac{\Delta C}{\Delta w} = \frac{\$48 - \$32}{180 - 100} = \frac{16}{80} = 0.2$$

We have $C = b + 0.2w$
Substitute $w = 100$, $C = \$32$: $32 = b + 0.2(100)$

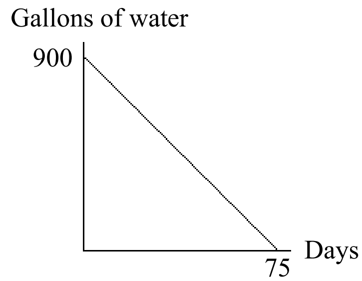
$$32 = b + 20$$
$$b = 12$$

Therefore $C = 12 + 0.2w$.

(b) The slope is \$0.20 per kg,
which is the monthly rate that the service charges for waste collection.

(c) The vertical intercept is (0, \$12).
When no waste is collected, the service charges a fixed charge of \$12.

29. (a) Since we start with 900 gallons of fresh water, the vertical intercept is (0, 900).
Each day we lose 12 gallons of water so the equation is $f(t) = 900 - 12t$.



(b) (i) $f(0) = 900$.
Initially we have 900 gallons of water.

(ii) To find $f^{-1}(0) = t$, we must find the time t when the team has 0 gallons of fresh water.

$0 = 900 - 12t$

$12t = 900$

$t = \frac{900}{12} = 75$

Thus $f^{-1}(0) = 75$ days.
It will take 75 days before the team has 0 gallons of water remaining.

30. (a) When we have zero U.S. dollars, we have zero shillings: the y-intercept is (0, 0).
(b) We need an equation for $y = f(x)$.
We first find the slope of the function.
The function is increasing so we expect a positive slope.
One way is to find the slope is to compute Δy and Δx.

Δx	U.S. dollars	Shillings	Δy
	$1.00	2200	
$1.50	$2.50	5500	3300
$0.50	$3.00	6600	1100

We want $\frac{\Delta y}{\Delta x} = \frac{3300\text{ shillings}}{\$1.50} = 2200$

Check that this is also the same as $\frac{1100\text{ shillings}}{\$0.50} = 2200$ shillings per U.S. dollar.

Since the y-intercept is (0, 0), the equation is $y = 2200x$.
Check: The equation passes through the point (1, 2200), as well as the other points in the table.
If $y = 4000$ shillings, then $4000 = 2200x$ so $x = \frac{4000}{2200} \approx \$1.82$. The trousers cost $1.82.
(Recall these are second-hand items in the Kampala market.)
(c) If we have $x = \$4.00$, then we can exchange it for $y = 2200x = 2200 \cdot 4 = 8800$ shillings, so we can afford the 8500 shilling coat.

31. (a) We know that when the price $p = \$10$, the number of customers N who will come to the park is 10,000. For each $1.00 increase in the entrance price p, the park would lose an average of 500 daily customers:

p	N
$10.00	10,000
$11.00	9,500
$12.00	9,000
$13.00	8,500
$14.00	8,000

(b) $N = f(p)$ is linear. When $\Delta p = \$1$, then $\Delta N = -500$.
The slope is $\frac{\Delta N}{\Delta p} = \frac{-500}{\$1} = -500$ and it passes through ($10, 10,000).
We have $N = b - 500p$. Substitute $p = 10$, $N = 10{,}000$:

$10{,}000 = b - 500(10)$

$10{,}000 = b - 5{,}000$

$b = 15{,}000$

p	N	$R = p \cdot N$
$10.00	10,000	$100,000
$11.00	9,500	$104,500
$12.00	9,000	$108,000
$13.00	8,500	$110,500
$14.00	8,000	$112,000

Therefore $N = f(p) = 15{,}000 - 500p$. *TIP:* Check the formula using the table feature of a grapher.

(c) If 10,000 customers pay \$10 each, the revenue is $10{,}000 \cdot \$10 = \$100{,}000$. Do this for each row to complete the table. Notice revenue increases due to the ticket price increase, although N decreases.

(d) In general, R is the product of the first two columns, so $R = p \cdot N$. Since $N = 15{,}000 - 500p$, we have $R = p \cdot N = p \cdot (15{,}000 - 500p)$. Check the formula using the table feature of a grapher.

(e) To find the N-intercept of $N = 15{,}000 - 500p$, set $p = 0$. By inspection it is (0, 15,000). Interpretation: If the tickets were free, Ruby's Pier would have 15,000 customers.
To find the p-intercept of $N = f(p)$, set $N = 0$ and solve for p:

$$N = 15000 - 500p$$
$$0 = 15000 - 500p$$
$$500p = 15000$$
$$p = 30$$

The p-intercept is (30, 0).
Interpretation: If the ticket price was \$30, no customer would purchase one.

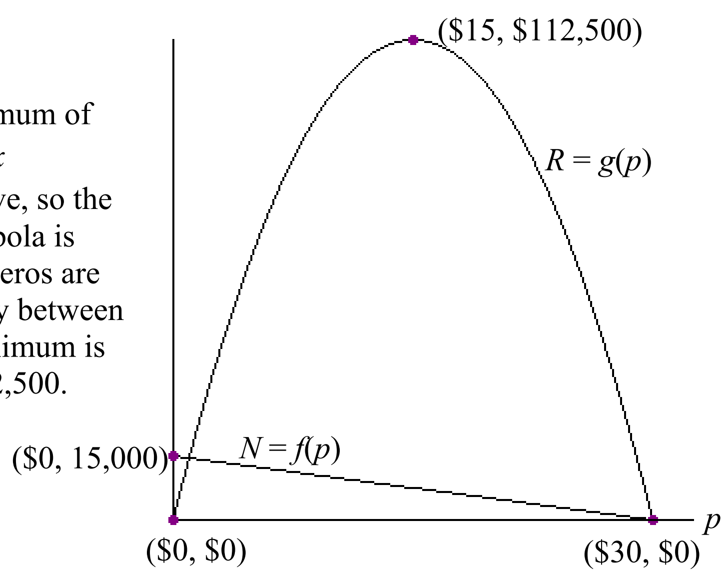
(f) Find any p-intercepts by solving $R = p \cdot (15{,}000 - 500p) = 0$
Set each factor equal to 0: we know $R = 0$ when $p = 0$ and $15{,}000 - 500p = 0$.
From part **(e)**, $15{,}000 - 500p = 0$ when $p = 30$ so the p-intercepts are (0, 0) and (30, 0).
Interpretation:
(0, 0): If the tickets were free,
there would be no revenue (even though 15000 customers would come).
(30, 0): If the tickets were \$30, there would be no revenue (since no customers would buy them.)

To find all the R-intercepts, set $p = 0$ in the equation $R = g(p) = p \cdot (15{,}000 - 500p)$.
$R = g(0) = 0 \cdot 15{,}000 = 0$.
The only R-intercept is the point (0, 0), interpreted previously.

(g) \$15 since the maximum is at (15, 112,500).
There are several strategies to get the maximum of
$R(x) = x(15{,}000 - 500x) = -500x^2 + 15000x$

1. The coefficient of the x^2 term is negative, so the parabola is concave down. Since a parabola is symmetric about its maximum, and its zeros are at $x = 0$ and 30, the maximum is midway between at $x = 15$. The y= coordinate of the maximum is $R = g(15) = 15 \cdot (15{,}000 - 500 \cdot 15) = 112{,}500$.
2. Use the maximum feature.
3. Use the table feature.

(h) See the graph at the right.



32. The formula for the power function is $y = 5000x^{-3}$.

For $y = kx^p$, we have $-40 = k(-5)^p$ and $5 = k(10)^p$. Take ratios.

$$\frac{-40}{5} = \frac{k \cdot (-5)^p}{k \cdot 10^p}$$

$$-8 = \left(\tfrac{-5}{10}\right)^p$$

$$-8 = \left(\tfrac{-1}{2}\right)^p$$

$$p = -3$$

We have $y = kx^{-3} = \dfrac{k}{x^3}$

Use (10, 5) to find k:

$y = \dfrac{k}{x^3}$ so solve $5 = \dfrac{k}{10^3}$

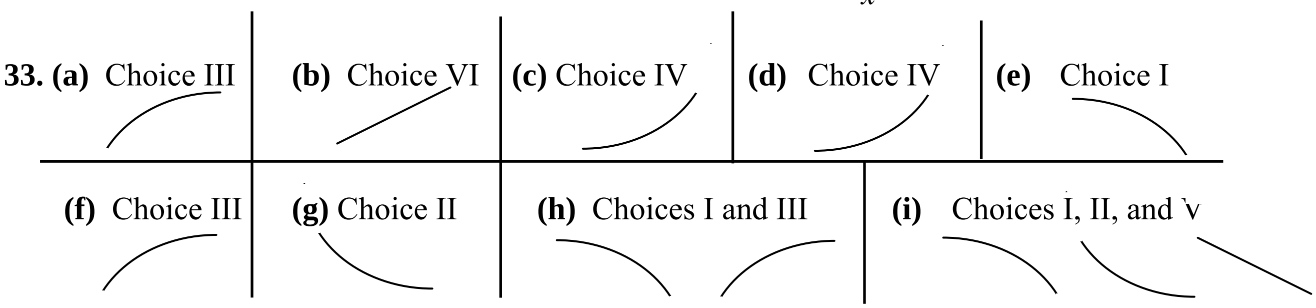
$k = 5000$

$y = 5000x^{-3}$

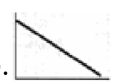
The formula for the power function is $y = 5000x^{-3}$ or $y = \dfrac{5000}{x^3}$.

33. (a) Choice III **(b)** Choice VI **(c)** Choice IV **(d)** Choice IV **(e)** Choice I

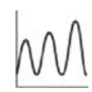
(f) Choice III **(g)** Choice II **(h)** Choices I and III **(i)** Choices I, II, and V



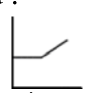
34. a. Choice II. The train's speed slows to a stop (speed is 0).



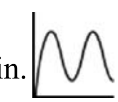
b. Choice I. The swing moves away from the ground, then toward it, and repeats, but each time gets higher from the ground .



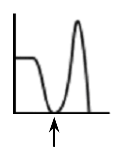
c. Choice III First her speed is constant.
When she runs, her speed increases.



d. Choice II The ferris wheel car climbs to its highest point, then descends, then climbs again.



e. Choice III As the child climbs up the slide her speed is steady and constant.
When she stops at the top of the slide, her speed is 0.
Once she slides down her speed increases, exceeding the speed she had when she was climbing the slide.
At the bottom of the slide, her speed is 0 when she stops.



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35. (i) $P(t) = 300 - 2t$ is Choice **F** since $300 - 2t = 250$ when $t = 25$.

The population starts at 300 and has dropped to 250 after 25 years.

It is not Choice **A** since, even though $P(t) = 300 - 2t$ declines at a constant rate, $P(t)$ becomes 0 in 150 years, not 15.

(ii) $Q(t) = 300e^{0.02t}$ is Choice **C**.

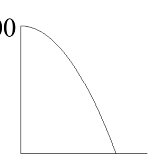
The population, which began at 300, is growing at the continuous rate of 2 percent each year.

(iii) $R(t) = 300(0.98)^t$ is Choice **H**.

The population, originally at 300, has been decreasing at the annual rate of 2 percent.

(iv) $S(t) = -\frac{4}{3}t^2 + 300$ is Choice **G**.

The population, which began at 300, decreases faster and faster.



36. (i) $f(x) = \dfrac{5x^2 - 5}{8000x - 80}$

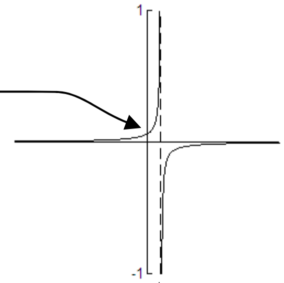
***x*-intercepts:** (1, 0) and (−1, 0).

The *x*-intercepts occur when $y = f(x) = 0$, which is when the numerator $5x^2 - 5 = 0$. Solve $5x^2 = 5$

$$x^2 = 1$$
$$x = 1, -1$$

***y*-intercept:** $(0, \frac{1}{16})$.

Set $x = 0$. $y = \dfrac{5x^2 - 5}{8000x - 80} = \dfrac{5(0)^2 - 5}{8000(0) - 80} = \dfrac{-5}{-80} = \dfrac{1}{16}$



vertical asymptote: $x = 0.01$

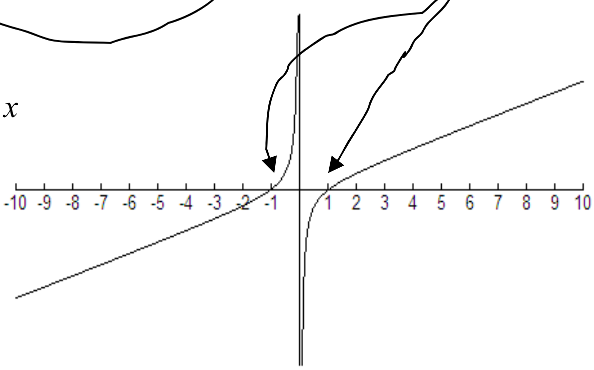
Solve $8000x - 80 = 0$

$$8000x = 80$$
$$x = \tfrac{80}{8000} = 0.01$$

horizontal asymptote: None.

As $x \to \infty$, $f(x) = \dfrac{5x^2 - 5}{8000x - 80} \approx \dfrac{5x^2}{8000x} = \dfrac{5x}{8000} = \tfrac{1}{1600}x$

For large values of *x* the function looks very much like the linear function $y = \frac{1}{1600}x$.



Note: Graphs are not expected, but shown here to confirm the algebraic reasoning. None of these graphs show the complete behavior of the function by themselves.

(ii) $f(x)=\dfrac{5x^2+5}{8000x^2-80}$

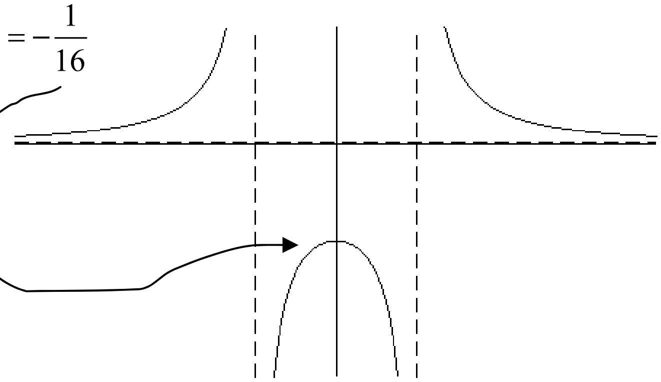
***x*-intercepts:** None

The x-intercepts occur when $y = f(x) = 0$, which is when the numerator $5x^2+5=0$.

However, $5x^2+5$ is never 0, so there are no x-intercepts.

***y*-intercept:** $(0, -\frac{1}{16})$.

Set $x = 0$. $y=\dfrac{5x^2+5}{8000x^2-80}=\dfrac{5(0)^2+5}{8000(0)^2-80}=\dfrac{5}{-80}=-\dfrac{1}{16}$



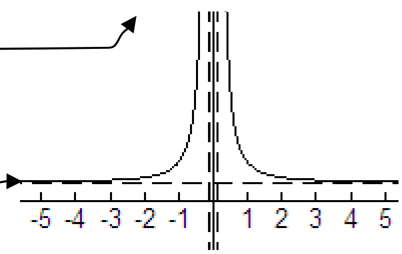
vertical asymptotes: $x=\frac{1}{10}$, $x=-\frac{1}{10}$

Set the denominator equal to 0.

$$8000x^2-80=0$$
$$8000x^2=80$$
$$x^2=\tfrac{80}{8000}=\tfrac{1}{100}$$
$$x=\pm\tfrac{1}{10}$$

horizontal asymptote: $y=\frac{1}{1600}$

As $x\to\infty$, $f(x)=\dfrac{5x^2+5}{8000x^2-80}\approx\dfrac{5x^2}{8000x^2}\to\dfrac{5}{8000}=\dfrac{1}{1600}$



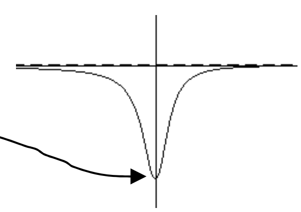
(iii) $f(x)=\dfrac{5x-5}{8000x^2+80}$

***x*-intercept:** $(1, 0)$

The x-intercept occurs when $y = f(x) = 0$, which is when the numerator $5x-5=0$ or when $x=1$.

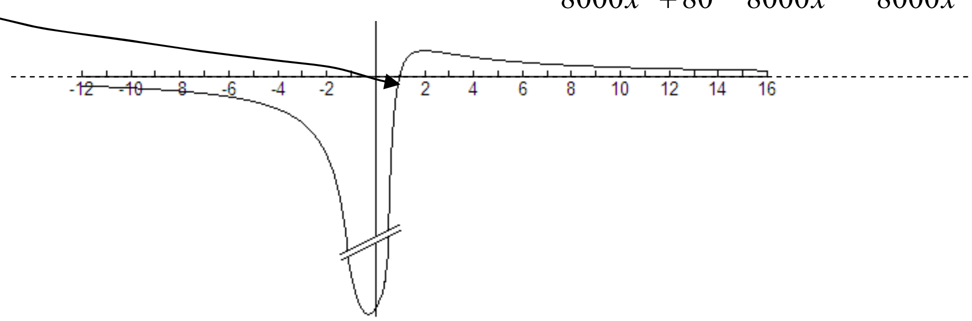
***y*-intercept:** $(0, -\frac{1}{16})$.

Set $x = 0$. $y=\dfrac{5x-5}{8000x^2+80}=\dfrac{5(0)^2-5}{8000(0)+80}=\dfrac{-5}{80}=-\dfrac{1}{16}$



vertical asymptotes: None, since $8000x^2+80$ is never 0.

horizontal asymptote: $y = 0$ since as $x\to\infty$, $f(x)=\dfrac{5x-5}{8000x^2+80}\approx\dfrac{5x}{8000x^2}=\dfrac{5}{8000x}\to 0$.



37. (a) $1000e^{(0.05 \cdot 1)} \approx \$1051.27$

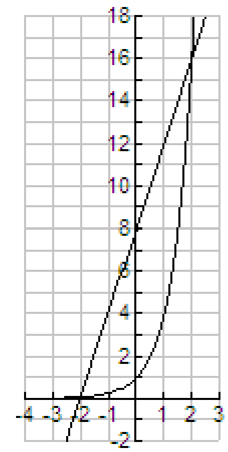
(b) No since log 10 = 1.

(c) Simplify as much as possible: $e^{\ln x^2 + \ln 5} = e^{\ln 5 + \ln x^2} = e^{\ln 5} e^{\ln x^2} = 5x^2$

(d) First, the graph of $f(x)$ is shifted to the left by 4 units, then (second) it is stretched vertically by a factor of 2, and then (third) it is shifted upward by 3 units.

(e) $x = 2$ is a solution to the equation $4x + 8 = 4^x$ since

$$4x + 8 = 4^x$$
$$4(2) + 8 = 4^2$$
$$8 + 8 = 16$$



To solve $4x + 8 > 4^x$, we must find all solutions to $4x + 8 = 4^x$, which can only be solved graphically or numerically.
The solutions are $x = -1.98403$ and $x = 2$.

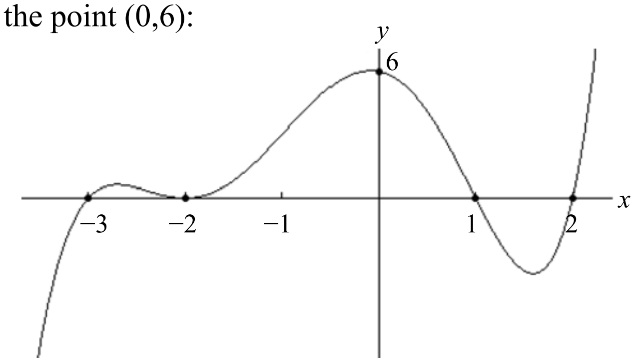
From the graph, the solution to $4x + 8 > 4^x$ are the values of x when the graph of $y = 4x + 8$ is ***above*** the graph of $y = 4^x$, which is $-1.98403 < x < 2$

38. Choice **C**. $h(x) = x^3$ The domain and range are all real numbers.

39. (a) The polynomial has formula $y = \frac{1}{4}(x-2)(x-1)(x+3)(x+2)^2$

Because the function has single zeros at –3, 1, and 2 and a double zero at –2 we can write $y = a(x-2)(x-1)(x+3)(x+2)^2$ Now substitute the point (0,6):

$$\left.\begin{array}{l} x = 0 \\ y = 6 \end{array}\right\} y = a(x-2)(x-1)(x+3)(x+2)^2$$
$$6 = a(-2)(-1)(3)(2)^2$$
$$6 = 24a$$
$$a = \frac{6}{24} = \frac{1}{4}$$



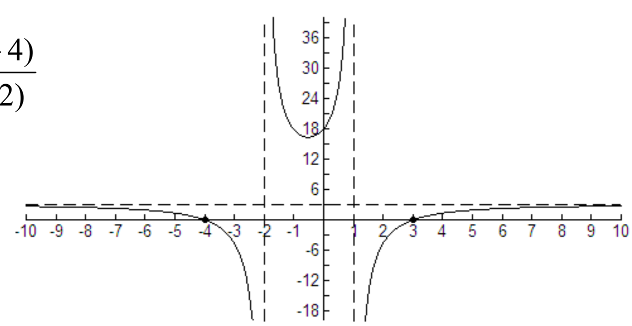
Therefore, the polynomial is $f(x) = \frac{1}{4}(x-2)(x-1)(x+3)(x+2)^2$

To find $f(3)$, let $x = 3$: $f(3) = \frac{1}{4}(3-2)(3-1)(3+3)(3+2)^2 = \frac{1}{4}(1)(2)(6)(5)^2 = 75$

You could also use the table feature of a graphing calculator. Choice **B**.
Important: You should check with a graphing calculator to be sure that the function is correct.

(b) The rational function has the formula $y = \dfrac{3(x-3)(x+4)}{(x-1)(x+2)}$

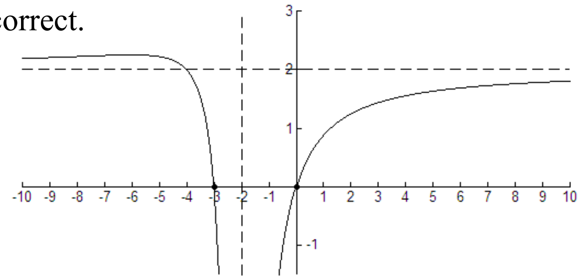
Because the zeros of the function are 3 and –4, the factors of the numerator are $(x+3)(x-4)$, since the function is 0 when the numerator is 0.



Since the vertical asymptotes are $x = -2$ and $x = 1$, the factors of the denominator are $(x-1)(x+2)$. (The vertical asymptotes are found where the denominator is 0 and the numerator is not).

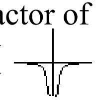
So we can write $y = \dfrac{a(x-3)(x+4)}{(x-1)(x+2)}$.

Since the horizontal asymptote is $y = 3$ and it is found by the ratio of the leading terms, we must have $a = 3$. Therefore the function must be $y = \dfrac{3(x-3)(x+4)}{(x-1)(x+2)}$. Check that when $x = 0$, you have $y = 18$. Use a table feature to find Choice **D** is correct.



(c) The rational function has the formula $y = \dfrac{2x(x+3)}{(x+2)^2}$.

Because the zeros of the function are 0 and –3, the factors of the numerator are $x(x+3)$, since the function is 0 when the numerator is 0.

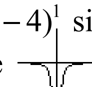
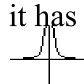
There is one vertical asymptote at $x = -2$, so $(x+2)$ is a factor of the denominator. However, the short run behavior near this asymptote looks like $y = k/x^2$ () so the factor must have a power of 2.

We can write $y = \dfrac{ax(x+3)}{(x+2)^2}$. Since the horizontal asymptote is $y = 2$, we must have $a = 2$.

Note: $y = \dfrac{ax(x+3)}{(x+2)^2} \approx \dfrac{ax^2}{x^2} = a$ as $x \to \pm\infty$ so $a = 2$.

Therefore, the rational function has the formula $y = \dfrac{2x(x+3)}{(x+2)^2}$. Choice **D** is correct.

40. The equation is $y = \dfrac{8(x-4)}{(x-2)^2}$

Because there is a horizontal asymptote of $y = 0$, the degree of the numerator is less than the degree of the denominator. The numerator has a factor of $(x-4)^1$ since it has a single zero. Because the short run behavior near the vertical asymptote looks like  or  the lowest degree possible for the denominator must be 2. So it has a factor of $(x-2)^2$. It has the form $y = \dfrac{a(x-4)}{(x-2)^2}$, and we can find a if we use the fact that when $x = 0$, $y = -8$:

$$-8 = \frac{a(0-4)}{(0-2)^2}$$

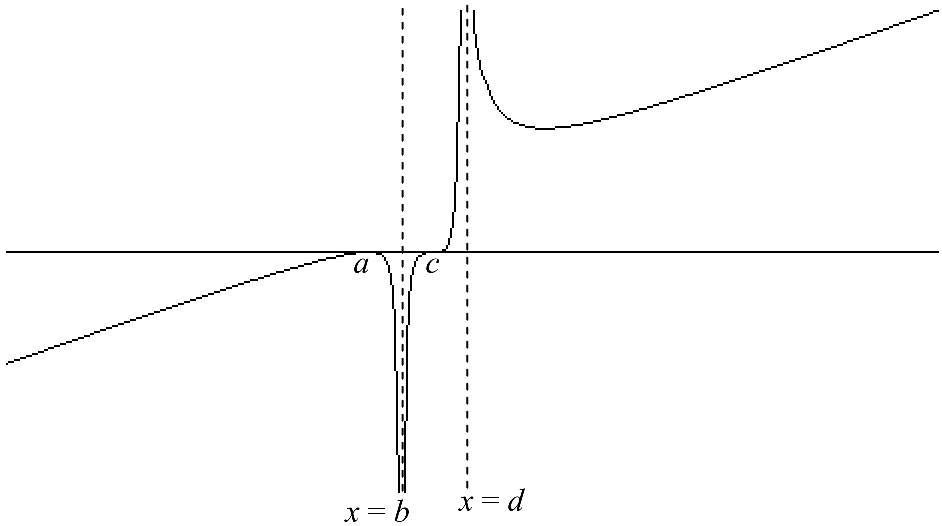
$$-8 = \tfrac{-4}{4}a$$

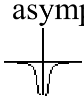
$$a = 8$$

So $f(x) = \dfrac{8(x-4)}{(x-2)^2}$**.** To find $f(3)$, we let $x = 3$ and find y. $f(3) = \dfrac{8(3-4)}{(3-2)^2} = \dfrac{8(-1)}{1} = -8$

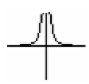
Choice **B.**

41. The degree of the factor $(x-a)$ must be even since there is a bounce at the zero.



The degree of the factor $(x-b)$ must be even since the vertical asymptote appears as  near b.

The degree of the factor $(x-c)$ must be 3, 5, … since there is a chair at the zero.

The degree of the factor $(x-d)$ must be even since the vertical asymptote appears as  near d.

The long run behavior is the same as the power function $y = kx$, so the degree of the numerator must be one more than the degree of the denominator. Therefore, it must be Choice **B**.

42. I. C $y = B - Ax$ since it has a positive y-intercept (B) and slope is negative ($-A$).

II. C $y = \log(x + A)$ since it is a shift of $y = \log x$ to the left A units. (Its vertical asymptote is at x = $-A$.)

III. A $y = |x - A|$ since it is a shift of $y = |x|$ to the right A units. (Its minimum is when $x = A$.)

IV. C $y = A(x+B)^2 - C$ since the x-and y-coordinate coordinates of the vertex are negative and the parabola is concave up.

V. C $y = -A(x+B)^5 + C$ since it is a vertical reflection of $y = x^5$ combined with a horizontal shift to the left and a vertical shift up.

VI. D $y = (1/A)^x$ since it is exponential decay.

VII. C $y = \dfrac{A(x+B)}{x-C}$ since its vertical asymptote is x = C with C positive, it has a horizontal asymptote $y = A$ with A positive, and a negative zero (at $-B$).

VIII. A $y = \dfrac{A}{(x-B)^2} - C$ since it is a shift of $y = \dfrac{A}{x^2}$ to the right B units and down C units.

43. slope: $-\frac{2}{3}$ point: (60, 30)

$$\left.\begin{array}{l} x=60 \\ y=30 \end{array}\right\} \quad y=-\frac{2}{3}x+b$$

$$30=-\frac{2}{3}(60)+b$$

$$30=-40+b$$

$$b=30+40=70$$

Therefore the equation is $y=-\frac{2}{3}x+70$.

We were asked for the ***x*-intercept**. Set $y = 0$ and solve:

$$0=-\tfrac{2}{3}x+70$$

$$\tfrac{2}{3}x=70$$

$$x=70\cdot\tfrac{3}{2}=105$$

The answer is Choice **A**.

TIP: You can narrow down the choices by making a rough sketch of the line.

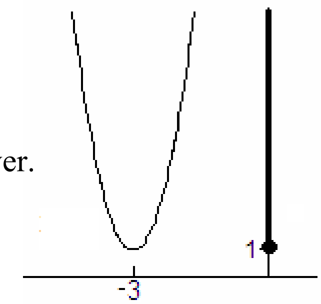
44. The range is the set of all possible values of y.

The range of the function $y=5x^2$ is all real numbers greater than equal to 0.

The function shown is a translation of $y=5x^2$ up 1, so the range is $[1,\infty)$.

Notice on the graph to the right, values of y begin at $y = 1$ and increase forever.

Choice **B**.



45.

$$\log_b\left(\frac{x^3y^2}{\sqrt{w}}\right)=\log_b x^3+\log_b y^2-\log_b\sqrt{w}$$

$$=\log_b x^3+\log_b y^2-\log_b w^{1/2}$$

$$=3\log_b x+2\log_b y-\tfrac{1}{2}\log_b w$$

Choice **C**.

46.

$$25^x=3^{600}$$

$$\ln 25^x=\ln 3^{600}$$

$$x\ln 25=600\ln 3$$

$$x=\frac{600\ln 3}{\ln 25}\approx 204.78$$

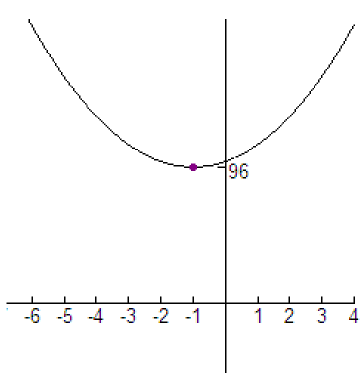
Choice **C**.

47. $(-1, 96)$ You can find the vertex by completing the square or use technology.

Choice **D**.

Note: If you use technology to find the minimum of the graph, you would **not** report points such as $(-1.000001, 96)$ or $(-0.9999983, 96)$.

The table feature, however, would show $(-1, 96)$.



$$y=4x^2+8x+100$$

$$=4(x^2+2x\qquad)+100$$

$$=4(x^2+2x+1-1)+100$$

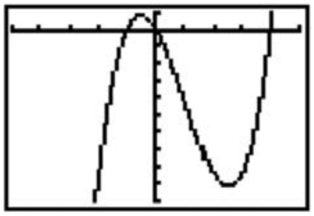
$$=4[(\underline{x^2+2x+1}-1)]+100$$

$$=4[(\underline{x+1})^2-1)]+100$$

$$=4\cdot(x+1)^2-4\cdot 1+100$$

$$=4(x+1)^2+96$$

48. Sketch a graph of the function.

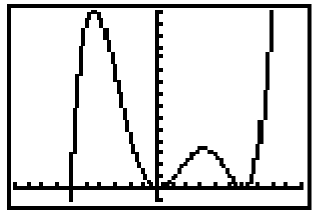


This suggests zeros at −1, 0, and 4:

$$7(x^3 - 3x^2 - 4x) = 0$$
$$7x(x^2 - 3x - 4) = 0$$
$$7x(x-4)(x+1) = 0$$
$$7x = 0 \;\|\; x = 4 \;\|\; x = -1$$
$$x = 0$$

Choice **C**.

49. Sketch a graph of the polynomial $f(x) = 9x^2(x+6)(x-6)^2$ by hand (or use a grapher, but it's difficult to find a window). Determine the values of x for which f is above or on the x-axis, which is $x \geq -6$.
Choice **C**.

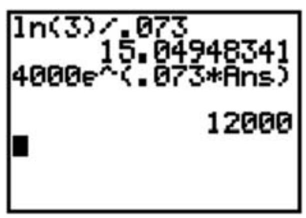


50. Solve $4{,}000e^{0.073t} = 12{,}000$.

$4000e^{0.073t} = 12{,}000$	Divide both sides by 4000 to get $e^{0.073t}$ all by itself.
$e^{0.073t} = 3$	Take natural logarithms of both sides.
$\ln e^{0.073t} = \ln 3$	Use the inverse property: $\ln e^{0.073t} = 0.073t$.
$0.073t = \ln 3$	Divide both sides by 0.073 to solve for t.

$$t = \frac{\ln 3}{0.073} \approx 15.05$$

Choice **D**.
TIP: Check by resubstituting:



51. The function $f(x) = \dfrac{1}{x^2}$ takes any input and returns the square of the reciprocal. We can replace x by a placeholder, such as an empty box, i.e. $f(\square) = \dfrac{1}{(\square)^2}$. If f takes the function $g(x) = \sqrt{x^2+4}$ as an input, then we have $f(\boxed{g(x)}) = \dfrac{1}{\left(\boxed{\sqrt{x^2+4}}\right)^2}$

$$= \frac{1}{x^2+4}$$

Choice **A.**

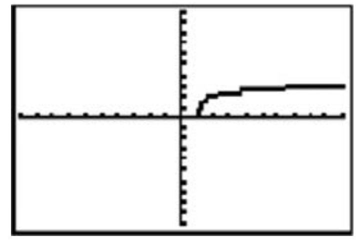
52. In general, the graph of $y = f(x) = ab^x$ increases for $b > 1$ and decreases for $0 < b < 1$ and has y-intercept $(0, a)$. The function $y = b^x$ is a special case, with $a = 1$. Therefore, Items I and III are correct. Choice **D**.

✓ I. It increases if $b > 1$
II. It decreases if $b < 0$
✓ III. It has y-intercept (0, 1) if $b > 0$.

53. The graph of $y = 2 + \log(x - 1)$ is a horizontal shift 1 unit to the right and a vertical shift 2 units up of the graph of $y = \log(x)$.

- Since the graph of $y = \log(x)$ has a vertical asymptote of $x = 0$, the graph of $y = 2 + \log(x - 1)$ has a vertical asymptote of $x = 1$.
- Since the domain of $y = \log(x)$ is the set of all real numbers $x > 0$, the domain of $y = 2 + \log(x - 1)$ is the set of all real numbers $x > 1$. Therefore it does **not** cross the x-axis at 1 and it never touches the y-axis.
- The graph of $y = 2 + \log(x - 1)$ passes through the point (2, 2): check: $x = 2, y = 2 \Rightarrow$

$$y = 2 + \log(x - 1)$$
$$2 = 2 + \log(2 - 1)?$$
$$2 = 2 + \log 1?$$
$$2 = 2 + 0?$$ **YES**



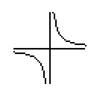
A graph of $y = 2 + \log(x-1)$ produced by technology in a standard window $-10 \le x \le 10$, $-10 \le y \le 10$ can look misleading!

- The range of the function $y = 2 + \log(x - 1)$ is all real numbers.

It is difficult for most technology to produce an accurate graph of a logarithm function. Don't be deceived by a misleading graph. Therefore Items I, III, and IV are correct. Choice **E**.

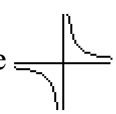
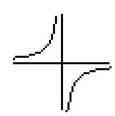
✓ I. increases for all values of x in its domain.
~~II. crosses the x-axis at 1~~
✓ III. never touches the y-axis
✓ IV. passes through the point (2, 2).

54. Since the vertical asymptote is $x = a$, the **denominator** must have $(x - a)$ as a factor. Since the function has a single zero through the origin (0, 0), the **numerator** must be 0 when $x = 0$.

The short run behavior of the function near its vertical asymptote looks like  , requiring the factor in the **denominator to be raised to an odd power**.

The equation $y = \dfrac{x}{x - a}$ is the only choice which meets these three criteria. Choice **C**.

55. As $x \to \infty$ or $x \to -\infty$, $f(x) = \dfrac{2ax}{(x-a)^2} \approx \dfrac{2ax}{x^2} = \dfrac{2a}{x}$.

In other words, the graph of $y = \dfrac{2ax}{(x-a)^2}$ and the graph of $y = \dfrac{2a}{x}$ have the same long run behavior. The graph of $y = \dfrac{2a}{x}$ has end behavior which looks like  or 

(depending on whether a is positive or negative).
In either case, as $x \to -\infty$ or as $x \to \infty$, the function approaches 0
The horizontal asymptote is $y = 0$. Choice **D**.

56. Solve $2.1 = -\log[H+]$. If we let $[H+] = x$, we have $-2.1 = \log x$.
By definition of the logarithm, $x = 10^{-2.1}$ or 0.008. Choice **B**.

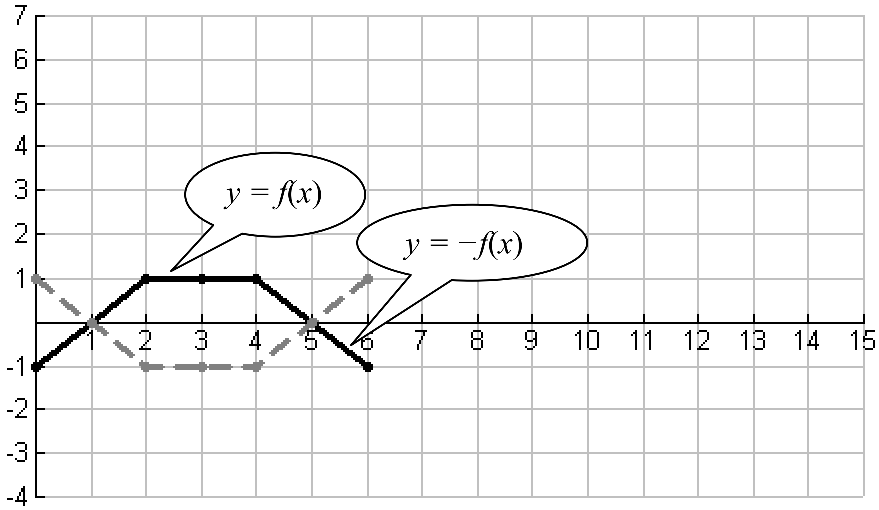
57. To solve $\ln 2x^3 = 5$, exponentiate both sides to base e: We have $e^{\ln 2x^3} = e^5$ or $2x^3 = e^5$. Choice **D**.

58. To solve $2x^3 = e^5$ (from Question 57), first divide by both sides by 2: we have $x^3 = \frac{1}{2}e^5 = \dfrac{e^5}{2}$.

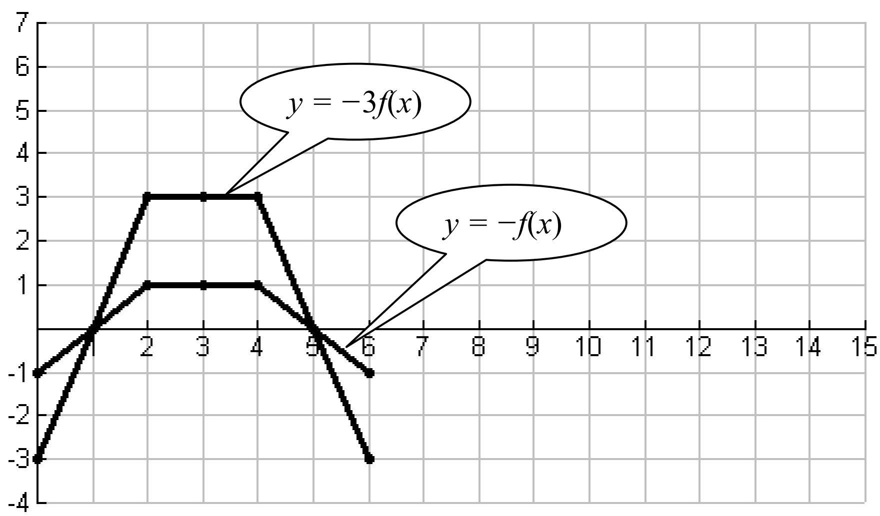
Then take the cubed root of both sides to get Choice **C**.

59. Choice **E**. $g(x) = -3f(x-8)+3$

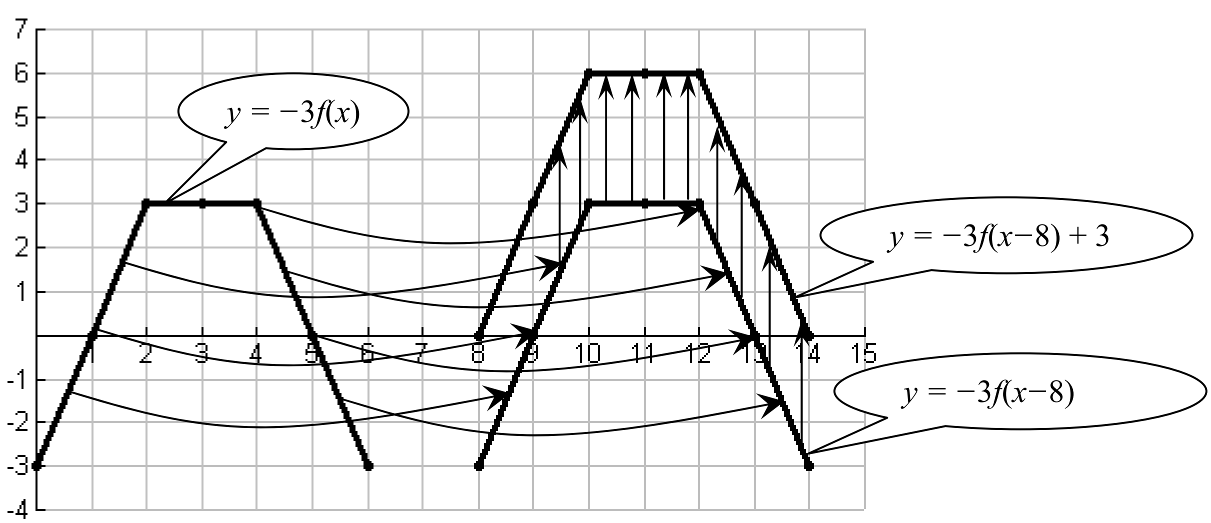
First reflect the graph of $y = f(x)$ vertically about the x-axis.



Second, stretch the graph of $y = -f(x)$ by a factor of 3.



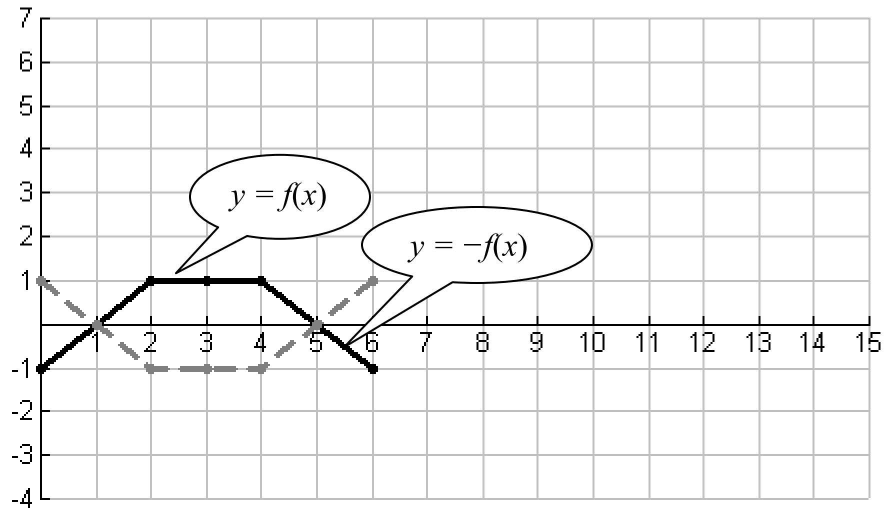
Third, shift the graph of $y = -3f(x)$ to the right 8 units. Finally shift the graph of $y = -3f(x-8)$ up 3.



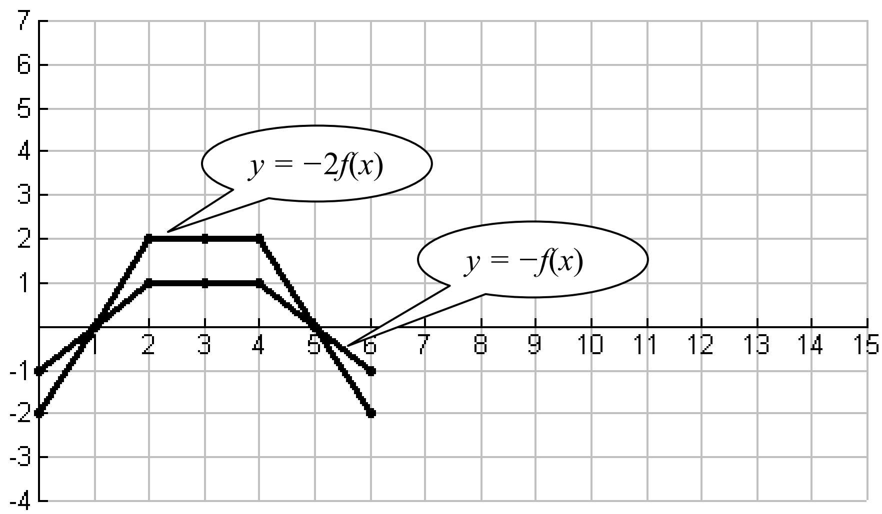
The equation of $g(x)$ is $g(x) = -3f(x-8) + 3$. Choice **E**.

60. Choice **C**. $h(x) = -2f(x) + 5$

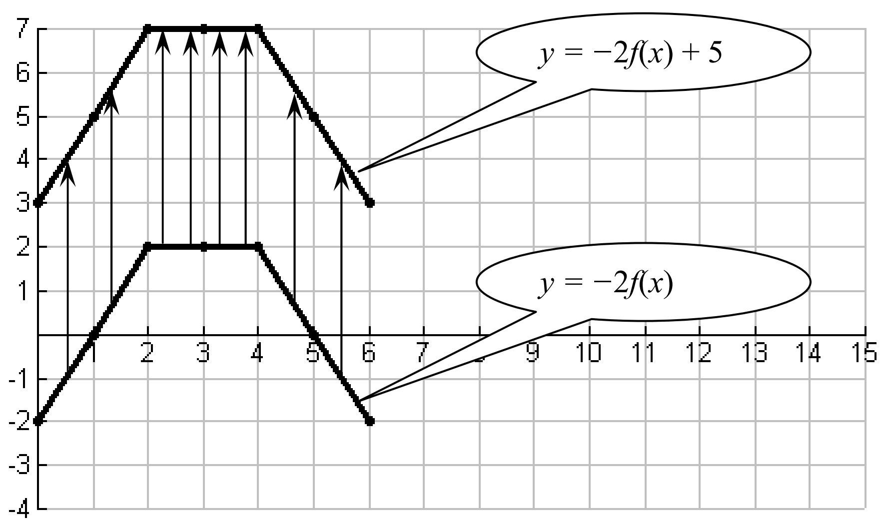
First reflect the graph of $y = f(x)$ vertically about the x-axis.



Second, stretch the graph of $y = -f(x)$ by a factor of 2.



Finally shift the graph of $y = -2f(x)$ up 5 units.



The equation is $h(x) = -2f(x) + 5$ Choice **C**.