

Title: Asymmetric Rhythms and Tiling Canons

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Abstract: Anyone who listens to rock music is familiar with the repeated drumbeat—one, two, three, four—based on a four-beat measure. However, other popular music (jazz, Latin, African) has different characteristic rhythms. Although much of this music is based on the four-beat measure, some instruments play repeated patterns that are not synchronized with the beat, creating *syncopation*—an exciting tension between different components of the rhythm. This paper is concerned with classifying and counting rhythms that are maximally syncopated in the sense that, even when shifted, they cannot be synchronized with the division of a measure into two parts. In addition, we discuss rhythms that cannot be aligned with other even divisions of the measure. Our results have a surprising application to rhythmic canons. A *canon* is a musical figure produced when two or more voices play the same melody, with each voice starting at a different time; in a *rhythmic canon*, rhythms, and not necessarily melodies, are duplicated by each voice. A rhythmic canon *tiles* if there is exactly one note onset in some voice on each beat. Upon mapping beats to integers, a rhythm forms a tiling canon if and only if its rhythmic motif and sequence of voice entries correspond to sets A and B forming a *tiling of the integers*—that is, a finite set A of integers (the tile) together with an infinite set of integer translations B such that every integer may be written in a unique way as an element of A plus an element of B . Although many have studied this problem, the complete classification of such tilings is an open question.

RACHEL W. HALL received a BA in Ancient Greek from Haverford College and a PhD in mathematics from the Pennsylvania State University. Her research interests are mathematical music theory and ethnomathematics. She is on the editorial boards of *Music Theory Spectrum*, *Journal of Mathematics and Music*, and *Journal of Mathematics and the Arts*. As a member of the folk trio Simple Gifts since 1995, she has toured throughout the Mid-Atlantic and released three albums. She plays the English concertina, piano, and (occasionally) tabla.