

**COMMENTS ON:  
REAL CONGRUENCE OF COMPLEX MATRIX PENCILS AND  
COMPLEX PROJECTIONS OF REAL VERONESE VARIETIES**

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1. ERRATA

The following typos appear in the published paper, [C<sub>2</sub>].

Line 3 of the Proof of Theorem 2.13: should read “As in the proof of” . . . .

The formula for  $\tau$  in Example 8.5 at the top of page 82 should read:

$$\tau : (z_1, iz_1, z_3, z_4)^T \mapsto (-(z_1 + z_3 + iz_4), -i(z_1 + z_3 + iz_4), z_3, z_4)^T$$

2. UPDATES

In addition to reference [19] by Morley and Morley, there is another book which considers quadratically parametrized real curves in the complex plane, as in Section 4. [D] arrives at essentially the same classification as that in Section 4, and goes into some detail on the relationship between the complex coefficients and the geometry of the curve.

The Veronese surface and its projections, including Steiner surfaces (Examples 2.8, 7.4) are discussed in [B] and [SR]. These books also describe ([B] §§II, IV; [SR] §III.1) the connection between projective equivalence classes of rational parametric maps and linear systems of varieties, which is briefly mentioned in Example 7.6. A complete list of the 13 types of maps from Example 7.6 is given by Degtyarev (reference [13]), and also appears on my web site.

Reference [5] will not appear under that title. However, due to a convenient editorial error, reference [5] is not cited in the paper anyway. The 4-manifold  $M$  in  $\mathbb{C}^5$  from Example 8.5 is considered in [C<sub>1</sub>] and, with an equivalent quadratic expression, in [C<sub>4</sub>].

References [6] and [8] have appeared as cited below: [C<sub>3</sub>], [CF].

3. CITATIONS

The article is cited in this paper: [W].

REFERENCES

- [B] A. BEAUVILLE, *Complex Algebraic Surfaces*, 2nd ed., LMSST **34**, Cambridge, 1996. MR 1406314 (97e:14045), Zbl 1001.32010.
- [C<sub>1</sub>] A. COFFMAN, *Enumeration and Normal Forms of Singularities in Cauchy-Riemann Structures*, dissertation, University of Chicago, 1997.

- [C<sub>2</sub>] A. COFFMAN, *Real congruence of complex matrix pencils and complex projections of real Veronese varieties*, *Linear Algebra and its Applications*, **370** (2003), 41–83. MR 1994320 (2004f:14026), Zbl 1049.14042.
- [C<sub>3</sub>] A. COFFMAN, *Analytic normal form for CR singular surfaces in  $\mathbb{C}^3$* , *Houston J. Math* (4) **30** (2004), 969–996. MR 2110245 (2006d:32048), Zbl 1074.32013.
- [C<sub>4</sub>] A. COFFMAN, *Analytic stability of the CR cross-cap*, *Pacific J. Math.* (2) 226 (2006), 221–258. MR 2247863 (2007j:32038), Zbl 1123.32018.
- [CF] A. COFFMAN and M. FRANTZ, *Möbius transformations and ellipses*, *The Pi Mu Epsilon Journal* (6) **12** (2007), 339–345.
- [D] R. DEAUX, *Introduction to the Geometry of Complex Numbers*, transl. from the revised French ed. by H. EVES, Ungar Pub. Co., New York, 1957. MR 0086745 (19,236f), Zbl 0077.14502.
- [SR] J. G. SEMPLE and L. ROTH, *Introduction to Algebraic Geometry*, Oxford, 1985. MR 0814690 (86m:14001), Zbl 0576.14001.
- [W] W. WATERHOUSE, *Real congruences of complex subspaces of  $2 \times 2$  symmetric complex matrices*, *Linear Algebra and its Applications* (2–3) **414** (2006), 502–505. MR 2214402 (2006j:15038), Zbl 1091.15033.

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