The branch and bound method was used to find the "best" number of sales regions and to reconfigure regional boundaries for the Variable Annuity Life Insurance Company (VALIC). The solution specified (1) an increase in sales regions from 16 to 25, and (2) the geographic customer base to be served by each region. Savings were estimated to be $8,833,000.
slightly different and relatively simple branch and bound model which takes into account fewer variables, but allows the number of territories to vary. The model was developed and employed for the Variable Annuity Life Insurance Company (VALIC), a Houston-based marketer to employees of not-for-profit organizations and governments at all levels.

Employees of not-for-profit organizations and of governments are, thanks to the Internal Revenue code, the only people eligible to participate in certain tax-sheltered investment and savings plans. Since published data abound on the distribution of such employees by state, it is a market reasonably easy to quantify on the basis of geographic segments. One can, in other words, say to a sales force, "There are 42,618 eligible buyers in Arkansas; go to it!"

An interesting question for VALIC in 1982 was how to organize that sales force, consisting of 336 individuals. At that time, there were 16 regions, each with a manager and an office. The regions had more or less simply evolved as combinations of 57 geographic segments — states or portions of states (for example, Southern California). Management was concerned about the extent to which market potential appeared unequal from one region to another. They were aware, however, that if regions were reconfigured arbitrarily regional managers' morale might be adversely affected. Such a situation is ripe for the application of management science.

The study of VALIC's problem began with a fixed "territory" (the US), a fixed number of salespersons, and the 16 regional headquarters (with associated fixed costs) already in place. We were asked (1) to determine the costs associated with the present 16-region configuration, (2) to determine the lowest cost solution in both number of regions and their geographic configuration, and (3) to calculate the cost savings if the change in configuration were to be adopted.

VALIC imposed three constraints:
(1) The number of regions should not decrease.
(2) The number should at most double.
(3) Disproportions in market potential should not be exacerbated.

However, the primary task was to improve profitability, not to equalize region-to-region market potential. It was also understood that the proportion of all VALIC salespersons assigned to a given region would continue to be a function of the proportion of US market potential within the region, and that the total number of salespersons would not change.

The Model

The problem presented suggested the trade-off between fixed and variable costs usually associated with combinatorial problems and amenable to solution with implicit enumeration techniques like branch and bound and specially designed heuristics. (See Khumawala [1972, 1973] for our experience with these techniques and heuristics to solve location problems.) In this situation, however, the trade-off was at first glance obscure. It was apparent that more regions would mean higher fixed costs — the costs for additional offices. However, the variable costs associated with fewer, larger regions — the
present configuration — were unclear.

We hypothesized, however, that the cost associated with fewer, and therefore larger, regions would be a cost of foregone sales caused at least in part by greater travel time. The hypothesis was tested, correlating number of VALIC participants (those who have purchased) per 1000 market potential in each of VALIC’s current regions with region size measured in total market potential. As expected, the number of participants per 1000 potential dropped as total market potential increased \((r = -0.4, p = 0.1, n = 16)\).

The model we constructed was, therefore, that total costs \((TC)\) of any configuration of regions would be the sum of:

- **FC**: fixed costs (office, manager, other overhead), which would increase directly with the number of regions,
- **OE**: operating expenses (a per-salesperson cost), unaffected by the number of regions, and
- **CFS**: cost of foregone sales (to be described below), affected by the distances within each region between its regional headquarters and its other major markets.

VALIC offered estimates of fixed costs per regional office, assumed for convenience to be identical at any possible location. They also estimated operating expenses per salesperson, again independent of location. They then suggested that a “breakpoint” of 150 miles between regional headquarters and largest market served be the key figure in calculating the cost of foregone sales. **CFS** was assumed to be a function of the proportion of all salespersons in any region who would service a market more than 150 miles from their region’s headquarters, given any hypothetical configuration.

Specifically, with the concurrence of VALIC, **CFS** was calculated as follows for any configuration:

1. We assume that the largest city in each state or portion of state was a potential regional office and also was the largest market in that geographic segment.

2. We measured the distance in miles from each of these 57 “centers” to each of the other 56 (for example, Baltimore to Washington, DC) if there was any likelihood that one could be served from the other.

3. To determine whether there was any such likelihood and thereby simplify the program, we set for each of the possible 57 x 56 combinations a cutoff point, based on the distance between them and on their combined population, those two numbers multiplied. The cutoff number used was the highest value for this product from the current VALIC configuration. Any combination exceeding the cutoff was assigned a prohibitively high **CFS** so as to effectively remove it from consideration.

4. For all other combinations, those not eliminated as infeasible by (3), above, we performed a series of calculations. Looking at any potential region the program calculated the proportion of its salespeople who would serve each “center” within it and noted whether they should be categorized as “distant” (> 150 miles from regional headquarters).

For each “distant” salesperson, the po-
tential region was assessed a CFS. It was calculated by assuming that the salesperson’s travel time replaced one call per week. Given VALIC’s estimates of its success ratio on calls and its net revenue from a success, a yearly per-person CFS was derived.

One can . . . say to a sales force, “There are 42,618 eligible buyers in Arkansas; go to it!”

An illustration may clarify the use of the program to assess the trade-off between CFS and FC in an attempt to minimize total costs. The current Minneapolis region included Minnesota, both Dakotas, and Nebraska and was served by six salespersons. Two of the six were “distant,” since the largest cities in the Dakotas and Nebraska are more than 150 miles from Minneapolis, and these three states contain one-third of the region’s market potential. Thus, the CFS for the current region was the per-person CFS (for illustration, $100,000) x 2. The total cost (TC) of the current region, using hypothetical figures, could therefore be calculated as follows:

\[
\begin{align*}
\$150,000 & \text{ (FC)} \\
48,000 & \text{ (per-person OE x 6)} \\
200,000 & \text{ (per-person CFS x 2)} \\
\$398,000 & 
\end{align*}
\]

This total could then be compared to the total for a hypothetical alternative. Suppose, for example, that Minnesota became a one-state region and Nebraska and the Dakotas became another region with its headquarters in Sioux Falls, South Dakota. If the distance from Sioux Falls to Omaha, Nebraska were less than 150 miles and also the distance from Sioux Falls to Fargo, North Dakota were less than 150 miles, placing another regional office in Sioux Falls would

1. Raise FC by $150,000 (adding an office),
2. Lower CFS by $200,000 (no “distant” salespeople),
3. Leave total operating expenses unaffected, and
4. Therefore lower TC by $50,000.

The branch and bound program implicitly or explicitly evaluated all feasible configurations using this kind of “reasoning.” As the example shows, it was designed to produce a solution in which fixed costs would increase somewhat, but the cost of foregone sales would decrease by far more, as smaller regions would make possible more hours of selling time.

Initially we formulated the problem as

\[
\text{Minimize} \quad \sum_{ij} C_{ij} X_{ij} + \sum_i F_i Y_i \quad (1)
\]

Subject to

\[
\sum_i X_{ij} = D_j \quad \text{for all } j \quad (2)
\]

\[
\sum_j X_{ij} \leq S_i Y_i \quad \text{for all } i \quad (3)
\]

\[
Y_i = 1 \quad \text{if } \quad X_{ij} > 0 \\
= 0 \quad \text{otherwise for all } i \quad (4)
\]

Where

\[
C_{ij} = \text{sum of variable costs (operating expenses and cost of foregone sales) relating to the } i^{th} \text{ regional office and } j^{th} \text{ geographic segment,}
\]

\[
F_i = \text{fixed costs of } i^{th} \text{ potential office,}
\]

\[
D_j = \text{market potential of } j^{th} \text{ geographic segment,}
\]
$S_i =$ capacity of the $i^{th}$ potential office,
$Y_i =$ decision variable indicating the utilization ($Y_i=1$) or nonutilization ($Y_i=0$) of the $i^{th}$ regional office,
$X_{ij} =$ decision variable denoting the amount of market potential of the $j^{th}$ segment to be served from the $i^{th}$ regional office.

As can be seen, the above model is the classical "capacitated facility location" problem. However, this model was further simplified because VALIC did not want to restrict any region to a specific size, so long as the ratio of largest and smallest market potential was within a limit, as discussed previously. Accordingly, constraint sets (3) were removed from the model, (that is, $S_i$ was set as a very large number for all $i$) and the resulting problem to be solved was an uncapacitated facility location problem.

It can easily be seen that the problem given by (1), (2), and (4) is a mixed integer programming problem. The branch and bound technique for it works as follows: The problem is first solved without the integer restrictions on the $Y_i$; the resulting solution provides a lower bound on the problem. Any feasible solution to the problem provides an initial upper bound. The problem is partitioned by selecting a $Y_i$ and constraining it to 1 and 0 respectively (referred to as branching) and resolved to provide a new lower bound. An upper bound results when a solution with all integer $Y_i$'s is reached. The process of branching (and as a result improving the bounds) is continued until the stage is reached that no more branching is possible. At this stage, the incumbent upper bound is the optimal solution.

The choice of this particular model was based on the following considerations:

1. The number of regions is fixed in many models, but a specific charge in our assignment was to increase the number of regions, once calculations showed an inverse relationship between region potential and market penetration. Our problem, therefore, was not to design $m$ sales territories from a total of $n$ sales coverage units, as described by Zoltners and Sinha [1983], but to determine the optimal $m$ while simultaneously setting boundaries.

2. Fixed costs need not be considered if $m$ is given, as described above. Fixed costs must be considered, however, once $m$ varies. Including such costs in the objective results in a mixed integer programming problem, amenable for solution with branch and bound.

3. The assumption that a sales call foregone (due to travel time) is a revenue loss — in other words, that there is a linear relationship between sales time and sales response — appeared to be justified for VALIC more than would be the case in many other selling situations. Nearly 70 percent of all calls result in sales; thus time is money to an unusual degree. For other kinds of selling situations where each call is a "long shot," assessing a cost of foregone sales for each foregone hour of selling would be far more difficult to justify.

Results

The first use of the program for VALIC was to apply it to VALIC's present regional configuration. The cost was calcu-
lated at $18,826,000. With the aid of the program we were able to determine that by closing one regional office and moving a few regional boundaries, VALIC could reduce total costs to $18,088,000 simply through a reduction in fixed costs and also in "distant" salespersons and consequently in cost of foregone sales. Far more interesting, however, was the cost difference when the number of regions was permitted to vary. The total cost for the resulting solution of 25 regions was $9,993,000, a saving of $8,833,000.

Not surprisingly, there were obstacles along the way. Left to its own devices, the program violated the constraint that dis-proportions in market potential should not be exacerbated; it created geographically compact but giant-in-market-potential regions in the Northeast. Recognizing that we would be increasing TC, we modified the CFS so that in certain regions a "distant" salesperson was defined as one serving a market 50 miles or more from the regional office rather than the general 150 miles. Given the travel time to leave a large Eastern city, the change seemed defensible: indeed such a trip might well take long enough to replace a sales call. In the general case, the alternatives are to affect Cu (as was done here) or to increase (Fr) fixed cost for a high-market-potential segment. The decision here to place the burden on cost of foregone sales was based on our assessment of external validity — that CFS would be affected whereas fixed costs might not.

A second problem was the program's uncaring attitude toward the location of current regional offices. Given a region containing Baltimore and Wilmington, Delaware, for instance, it was as likely to choose Wilmington for the regional office site as Baltimore — despite the fact that Baltimore now houses the current office. Therefore, a one-time "new office" cost was employed to stack the deck in favor of the status quo where the program would otherwise have been indifferent.

**Limitations**

Limitations to the method presented here should, of course, be recognized. There are two: (1) the computational difficulties with a high number of potential offices, and (2) the possibility of split geographic segments.

The "too many potential offices" possibility arises when m becomes very large (say 60 or more) so that the optimizing code using the branch and bound algorithm presents computational difficulties. This would be especially true in the case when capacity constraints on the size of the region to be served (in market potential) are imposed on the problem.

... it created geographically compact but giant-in-market-potential regions in the Northeast.

The difficulty arises because the total number of combinations that need to be evaluated to produce optimality (either explicitly or implicitly) has an exponential relationship with m, specifically $2^m$. On the other hand, the relationship of the problem with respect to n, the total number of segments to be allocated, is linear. Hence a large number of such
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segments adds very little to the computational difficulty of problem solutions. It should be noted that in the uncapacitated case each segment is served in full from the cheapest region: that is, $X_{ij} = 1$ in the solution; hence separate integer requirements on the $X_{ij}$ in the problem are not needed.

To overcome the computational difficulty of branch and bound, we suggest the use of heuristic procedures for both the uncapacitated and capacitated problems; such procedures are outlined in Khumawala [1973] and [1974], respectively. Our experience is that these procedures have worked fairly well in practice, both in terms of solution quality and computational time and storage.

However, all things will be equal only if there is acceptance of the change by such individuals as the sales manager in the largest region, where market potential will decrease by 25 percent . . . .

The second potential limitation is the possibility of split segments, but it will arise only in the capacitated case. Since the uncapacitated case was employed in this project, however, there was no problem; all segments will be served from their respective “cheapest” (lowest $C_i$) regional office.

Conclusions

One year after they received the report, VALIC was described as beginning to phase in the plan. They reported that as they begin implementation, they are getting greater sales penetration but cannot yet state a dollar figure for incremental revenue. Implementation is expected to require three to five years in all.

Actual financial results will depend on the degree to which the suggested revisions are implemented. Sales region or sales territory configurations are notoriously areas in which managers are advised to move slowly and carefully, and VALIC clearly is aware of that advice. As regions increase in number from 16 to 25, it can be expected that corporate profitability will increase, all other things being equal. However, all things will be equal only if there is acceptance of the change by such individuals as the sales manager in the largest region, where market potential will decrease by 25 percent when the plan is implemented. Certainly a report can show a correlation coefficient which suggests to a regional manager that his or her market penetration can be expected to increase as the region’s size decreases; thus sales in the absolute sense may stay the same. But there is no reason to expect enthusiastic agreement.

Gradual changes seem more likely — and probably wise. After all, one thing the program was not asked to take into account was the CSM (Cost of Salesman-Misunderstanding).

References


A letter from Joe C. Osborne, Senior Vice President-Marketing, The Variable Annuity Life Insurance Company, states: "We expected that, because the project called for creation of regional offices, a three-to-five-year time frame would be appropriate for implementation. Since there are considerable fixed costs and complicated logistical considerations, a phased approach to the implementation of this plan is required.

Now that we have begun to implement the report, we find that we are getting greater sales penetration in those areas. In other words, the "foregone sales" identified in the report as a damper on revenue seems a realistic idea. It would be premature to offer a dollar figure for additional revenue that we attribute to our phased restructuring. However, we do plan to continue restructuring based on the recommendations in the report and to date have found it useful."